MODELING OF THE STRESS-STRAIN BEHAVIOR IN HARD SOILS AND SOFT ROCKS

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Abstract
The paper begins with a definition of hard soils and soft rocks (HSSR); this is followed by a short overview of the typical stress-strain behavior of HSSR. It is shown that in spite of the differences in the origin, type and strength of materials, similar stress-strain behaviors can be observed for different materials, ranging from soils to rocks. Based on this observed similarity a theoretical framework can be postulated, with which an appropriate constitutive model for HSSR can be formulated. This model includes the concepts of structure and destructurization as intrinsic material properties. A model named S_BRICK that takes into account the structure and destructurization has been developed and a comparison of this model's predictions with laboratory results is presented.

Keywords
hard soils and soft rocks, stress-strain behavior, constitutive modeling, S_BRICK, BRICK, structure, destructurization

1 INTRODUCTION

For a long time hard soils and soft rocks (HSSR) were treated as borderline cases in soil and rock mechanics; this was mostly due to the fact that their strength and stiffness properties usually exceeded the design requirements expected for soft soils. However, with the increasing number of large geotechnical projects executed in HSSR, a better understanding of their geomechanical behavior is needed so that they can be more accurately modeled.

First, a classification of HSSR will be presented; this will be followed by a description of their typical stress-strain behavior. It will be shown that regardless of their strength, these materials behave in a similar manner to soils. The main difference between HSSR and soft soils is in their structure, which is responsible for the higher strength and stiffness in HSSR.

A model called S_BRICK, which includes both structure and destructuring, will be briefly explained, and then a comparison between the S_BRICK model's predictions and the laboratory results on stiff North Sea clay will be presented. A comparison will also be carried out with a model that does not include structure. This model is called BRICK. It will be shown that the structure and its stability represent the key parameters that need to be accounted for in order to successfully model the behavior of HSSR.

2 A DEFINITION OF HARD SOILS AND SOFT ROCKS (HSSR)

From the practical point of view it is convenient to define HSSR according to their strength. There are several different classifications available, for example, ISRM [1], Bieniawski [2], BSI [3], and IAEG [4], to name just a small selection, which differ somewhat in terms of terminology and differ significantly in terms of defining the upper and lower limits of soils and rocks. IAEG [4], for instance, sets the limit for the uniaxial unconfined compressive strength (UCS), \( \sigma_c \), for “weak rock” at 15 MPa, BSI [3] sets it at 5 MPa, and ISMR [1] and Bieniawski [2] set the limit for weak rock at 25 MPa. Hawkins and Pinches [5] have proposed a classification for the entire range of geological materials, i.e., soils and rocks, based on the UCS for the upper limit and the undrained triaxial strength, \( c_u \), for the lower limit. The advantage of this classification is that it doubles each class of soil and rock and also acknowledges the continuum between soils and rocks. The classification is presented in Table 1.
### Table 1. The classification of soils and rocks according to their strength [5]

<table>
<thead>
<tr>
<th>Range</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_u &lt; 20$  kPa</td>
<td>Very soft soils</td>
</tr>
<tr>
<td>$20–40$ kPa</td>
<td>Soft soils</td>
</tr>
<tr>
<td>$40–80$ kPa</td>
<td>Firm soils</td>
</tr>
<tr>
<td>$80–160$ kPa</td>
<td>Stiff soils</td>
</tr>
<tr>
<td>$160–320$ kPa</td>
<td>Very stiff soils</td>
</tr>
<tr>
<td>$320–640$ kPa</td>
<td>Hard soils</td>
</tr>
<tr>
<td>$\sigma_c 1.25–2.5$ Mpa</td>
<td>Very weak rocks</td>
</tr>
<tr>
<td>$2.5–5$ Mpa</td>
<td>Weak rocks</td>
</tr>
<tr>
<td>$5–10$ Mpa</td>
<td>Moderately weak rocks</td>
</tr>
<tr>
<td>$10–50$ Mpa</td>
<td>Moderately strong rocks</td>
</tr>
<tr>
<td>$50–100$ Mpa</td>
<td>Strong rocks</td>
</tr>
<tr>
<td>$100–200$ Mpa</td>
<td>Very strong rocks</td>
</tr>
<tr>
<td>$\sigma_c &gt; 200$ Mpa</td>
<td>Extremely strong rocks</td>
</tr>
</tbody>
</table>

Geological materials classified as HSSR, which are written in bold in Table 1, represent an important fraction of all the geological materials in the geosphere, where most of construction takes place. They can be of different origin, ranging from igneous (decomposed and weathered granites or basalts, tuffs, etc.), to metamorphic (phyllites, weathered and decomposed gneisses and schists) to sedimentary origin (claystones, siltstones, flysh marls, etc.), and are the products of rock-forming, rock-altering and sediment-forming processes.

However, only using strength to distinguish between soils and rocks can be misleading when their engineering behavior is being considered. There are some instances when the behavior of rocks can be better described using the concepts of soil mechanics. When the frictional strength of discontinuities becomes comparable to the intact strength of the rock (Hyett and Hudson [6]), for example, at large depths, rocks can behave and fail in a plastic manner that is typical for soils. On the other hand, Picarelli and Olivares [7] describe the failure of stiff, highly fissured clay shales that fail on small-scale fissures that interconnect and form a discontinuity along which the material fails. Such phenomena are well described using the concepts of rock mechanics.

There is enough experimental evidence in the literature that demonstrates the conceptually similar stress-strain behavior of different geological material. Figure 1a, for example, shows oedometer compression and recompression curves for tests carried out on natural intact samples of three stiff clays and a marl (Burland et al. [8]), and Figure 1b shows oedometer results for tests carried out on three different clay shales (Bertuccioli and Lanzo [9]).

The compression curves of all the materials, ranging from stiff soils to marls and shales, show a similar compression behavior to that of soft soils, i.e., an initially stiff response until the normal compression line is reached, the beginning of isotropic hardening, and an increase of the state boundary surface with continuing compression, followed by a stiff response when unloading.

![Figure 1](image-url). Oedometer compression curves for a) three stiff clays and a marl [8], b) three different clay shales [9].
Figure 2 shows the results of isotropically consolidated drained triaxial tests on a Saint Vallier clay (Lefebre [10]) and a oolitic limestone (Elliot & Brown [11]). The tests labeled 1 were carried out at a low confining stress; the tests labeled 2 were carried out at an intermediate confining stress; and the tests labeled 3 were carried out at a high confining stress.

For both materials, when tested at a low confining stress (1), the results show a well-defined peak and a strain-softening behavior after the peak, with a dilating volumetric response. The test carried out at a high confining stress shows stiff behavior until the yield surface is encountered, from where the deviator stress slowly increases toward the critical state line. Note that the volumetric behavior is compressive. It is also important to note that regardless of the strength difference between the clay (soft soil) and the oolitic limestone (weak to moderately weak rock) the responses are similar and can be well described using the concepts of critical state soil mechanics.

Based on similar examples in the literature, Kavvadas [12] has proposed that the concepts of soil mechanics can be applied for the modeling of HSSR as long as the following two conditions are fulfilled:

1. the materials are significantly influenced by macrostructural features (large-scale discontinuities),
2. the influence of excess pore pressure is important.

This definition of HSSR is important because it opens up the possibility for the development of a constitutive model for the entire range of geological materials, from soft soils to soft rocks, within the theoretical framework of critical state soil mechanics.

3 STRUCTURE: A KEY PARAMETER FOR THE DEVELOPMENT OF CONSTITUTIVE MODELS FOR HSSR

It has been shown that in addition to important features like nonlinearity, state and stress history, a constitutive model has to include the effects of structure and destruc-
The influence of structure can be best observed when the behavior of a structured material is compared to the behavior of a reconstituted material. Structure is responsible for the increase of stiffness and strength in comparison to the reconstituted material, but the influence of structure is most clearly manifested in the larger state boundary surface (SBS) of the structured material. Leroueil and Vaugan [13] introduced the concept of structure-permitted space, which is shown in the \( v-p \) space in Figure 3, where \( v \) represents the specific volume and \( p \) represents the mean effective stress.

Figure 4 shows the state boundary surfaces for undisturbed, partly destructured and reconstituted Pappadai clay in a \( p/q \) diagram, normalized with the mean effective stress \( p_e^* \) taken at the isotropic reconstituted normal compression line using the same specific value as for intact clay (Cotecchia and Chandler [19]).

The influence of structure is clearly seen in the size of the state boundary surfaces, resulting in the higher strength of the undisturbed Pappadai clay in comparison to the partly destructured or reconstituted Pappadai clay.

Figure 3. Structure-permitted space by Leroueil and Vaugan [13].
Besides strength, structure also influences the stiffness across the entire range of deformations, with the most pronounced influence being in the range of small and very small deformations. Rampello and Silvestri [20] studied small strain stiffness in stiff Vallerica clay in the undisturbed and reconstituted states. They investigated the dependence of the elastic stiffness (denoted $G_0$) on the mean effective stress and the specific volume (Figure 5). For a given value of the mean effective stress or the specific volume, natural (undisturbed) clay has a higher value of elastic stiffness across the entire range of mean effective stress or specific volume.

According to Baudet [16] Vallerica clay has a stable structure; this can also be seen from Figure 5, where no tendency to converge can be observed for the shear moduli of the undisturbed and reconstituted clays. Similar results were obtained by Jovičić et al. [21],

![Figure 4](image)

**Figure 4.** Influence of structure on the state boundary surface of undisturbed, partly destructured and reconstituted Pappadai clay (Cotecchia and Chandler [19]).

![Figure 5](image)

**Figure 5.** Relationship between the elastic shear modulus $G_0$ and 
(a) the mean effective stress b) the specific volume (Rampello and Silvestri [20]).
who compared shear-stiffness degradation with strain for both reconstituted and intact stiff North Sea clays (Figure 6). They demonstrated that the influence of structure can be seen from the very small strains up to the point of failure.

A very important element of structure is its stability. We can see from Figure 3 that there is a tendency for the normal compression line of structured material to converge toward the normal compression line of the reconstituted material, which implies destructuring toward the reconstituted material. Destructuring caused by plastic straining is responsible for decreasing the state boundary surface, the strength and the stiffness.

Leroueil and Vaugan [13] have identified different yielding modes in natural materials. According to Leroueil and Vaugan [13] yielding can occur during shearing, compression and swelling, as shown in Figure 7. Similarly, destructuring can also be decoupled into shearing, compression and swelling.

![Figure 6](image1.png)

**Figure 6.** Comparison of shear-stiffness degradation with strain for two natural and reconstituted samples of stiff North Sea clay (Jovičič et al. [21]).

![Figure 7](image2.png)

**Figure 7.** Different modes of yielding and destructuring by Leroueil and Vaugan [13].
Destructuring during isotropic compression and swelling is governed purely by the volumetric component of the plastic strain. In the case of a normal compression stress path, the role of the deviator component in the destructuring is still not fully understood. However, it is reasonable to suspect that because the deviator component shows no tendency toward the state boundary surface, the influence of the deviator's plastic strain is negligible. During shearing, of course, the destructuring is governed by both the volumetric and deviator components of the plastic strain. It is also important to note that during swelling the destructuring of the stress paths can occur inside the state boundary surface, which was also shown by Leroueil and Vaugan [13].

\[ S_{\text{BRICK}} \text{ is given by the following two expressions:} \]

\[ \alpha_i, \omega \quad \text{initial values of structure parameters} \]
\[ \alpha_k, \omega_k \quad \text{final values of structure parameters} \]
\[ \alpha_i^{c,sh,sw}, \omega_i^{c,sh,sw} \quad \text{current values of structure parameters in compression (c), shear (sh) and swelling (sw)} \]
\[ \varepsilon_i^{pl}, \delta \varepsilon_i^{pl} \quad \text{volumetric and shear component of plastic strain (i=2-6)} \]
\[ \delta \varepsilon_v^{pl}, \delta \varepsilon_{v,i}^{pl} \quad \text{increment of volumetric and shear component of plastic strain (i=2-6)} \]
\[ x_1^{c,sh,sw}, y_1^{c,sh,sw} \quad \text{parameters that quantify influence of volumetric and deviatoric plastic strain of destructuring parameter } \alpha \]
\[ x_2^{c,sh,sw}, y_2^{c,sh,sw} \quad \text{parameters that quantify influence of volumetric and deviatoric plastic strain of destructuring parameter } \omega \]

all symbols are without units

The destructuring in S_BRICK is implemented separately, by introducing different parameters \( x_i^{c,sh,sw} \) and \( y_i^{c,sh,sw} \) for shearing (sh), compression (c) and swelling (sw). The decoupling of the plastic strain's influence on the volumetric and shear components and the introduction of the parameters \( x \) and \( y \), which quantify the rate of destructuring, gives the model an additional flexibility [24]. The full implementation of structure and destructuring requires the determination of an additional 16 parameters in total. Four of them (\( \alpha \), \( \alpha_k \), \( \omega \) and \( \omega_k \)) represent the structure and twelve \( (x_i, y_i)^{c,sh,sw} \) represent the destructuring of the structure in compression, swelling and shearing. It is reasonable to expect that not all types of destructuring are present for a dominant stress path, so it is very likely that the necessary total number of additional parameters can be as low as four. Accordingly, the destructuring is implemented in such a way that some model parameters that are not necessary or are not available can be omitted without hindering the behavior of the model. A more detailed formulation of the model is given by Vukadin [22], [23].

\[ \alpha_i^{c,sh,sw} = \alpha_k + (\alpha - \alpha_k) \exp \left[ -(x_i^{c,sh,sw} (\varepsilon_i^{pl} + \delta \varepsilon_i^{pl}) + y_i^{c,sh,sw} (\varepsilon_i^p + \delta \varepsilon_i^p)) \right] \]

\[ \omega_i^{c,sh,sw} = \omega_k + (\omega - \omega_k) \exp \left[ -(x_i^{c,sh,sw} (\varepsilon_i^{pl} + \delta \varepsilon_i^{pl}) + y_i^{c,sh,sw} (\varepsilon_i^p + \delta \varepsilon_i^p)) \right] \]
Vukadin et al. [23] have presented predictions of the S_BRICK and BRICK models on a conceptual level where the advantages of S_BRICK when modeling the structure and the destructurization were demonstrated. Here, a comparison of the modeled stress-strain behavior of stiff North Sea clay with the S_BRICK and BRICK models is presented. The stress-strain behavior of North Sea clay was investigated in the laboratory by Jovičić et al. [21]. Stress-path drained triaxial tests were carried out by investigating the strength and stiffness of reconstituted and natural samples at in-situ stresses and also when swelled back to effective stresses as low as 10 kPa [21]. In addition, undrained shear strength tests were also carried out.

All together, twelve different intact samples were investigated, taken from different depths, ranging between 15 and 70 m, and with undrained shear strengths ($\tau_{cu}$) ranging from 150 to 800 kPa. According to Table 1, North Sea clay can be classified as a very stiff soil to a very weak rock, depending on the section of clay being investigated [21]. The samples were taken from four different sections, based on an undrained profile with different stress histories and amounts of structure.

The input parameters for modeling the North Sea clay with BRICK were taken from Jovičić et al. [21], who in addition to other parameters took into account the material stress history as an input parameter. For each individual section of the clay a different history was modeled in such a way that the amount of over-consolidation was varied, iterated and then fixed for each clay section, so that the calculated and the measured undrained shear strengths coincide. It was concluded that for three clay sections, the iterated over-consolidation ratios were unrealistically high, which the authors [21] explained by the presence of structure in the clay, which was not accounted for with the BRICK model.

Table 2. Input parameters for the structure and destructurization for the modeled clay section with S_BRICK.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\alpha$</th>
<th>$\alpha_k$</th>
<th>$\omega$</th>
<th>$\omega_k$</th>
<th>$x_2$</th>
<th>$y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>0.85</td>
<td>0.85</td>
<td>1.2</td>
<td>0.6</td>
<td>800</td>
<td>900</td>
</tr>
</tbody>
</table>

The parameter $\alpha$ was determined from the critical state angle for the reconstituted clay obtained from triaxial shearing tests. The parameter $\omega$ was obtained by matching the small strain stiffness, and the parameter $\omega_k$, by matching the strength and stiffness at the critical state after the destructurization was finished. Due to the fact that no oedometric tests were available, only a destructurization during shearing was modeled, and the parameters $x_2$ and $y_2$ were obtained with a trial-and-error process so that a best fit across the entire range of deformation could be achieved.

All the numerical tests were carried out in the following steps:

- First, a stress history was numerically reproduced, prior to sampling as one-dimensional compression and swelling.
- Second, the amount of over-consolidation for the BRICK model was determined so that the undrained shear strength of the modeled clay coincided with the laboratory results. For the S_BRICK model a realistic stress history was modeled with the inclusion of the structure parameters $\alpha$ and $\omega$, so that the undrained shear strength was matched.
- Third, for both models the sampling and isotropic swelling or compression was modeled until the initial effective stress prior to shearing was reached.
- Finally, a numerical stress path was applied, which was similar to the stress path applied in the triaxial apparatus.

Two individual tests, TT6 and TT7, shown in Figures 8 and 9, were chosen as an example for a direct comparison between the observed and predicted stress-strain behaviors. Each figure shows separately the variation of the deviator stress $q$, the angle of the shearing resistance $\phi'$ and the volume strain $e_v$ with the axial strain $e_a$. In addition, a degradation of the secant shear modulus $G_s$ is shown against the logarithmic shear strain $e_s$, measured with local instrumentation. The sample TT6 was unloaded to an isotropic effective stress of 25 kPa with the OCR for BRICK equal to 406 and the OCR for S_BRICK equal to 120, while sample TT7 was unloaded to an isotropic effective stress of 50 kPa, with the OCR for BRICK equal to 203 and the OCR for S_BRICK equal to 60.
The numerically predicted behaviors by both models for the samples TT6 and TT7 are shown in Figures 7 and 8. It can be seen that the S_BRICK model more accurately reproduced the strength, stiffness and volumetric behavior for both samples. For sample TT6 the BRICK model overestimates the deviator stress and the mobilized frictional response and greatly underestimates the stiffness response and dilation in comparison with the S_BRICK model, whose prediction of the strength and stiffness behavior was very good. The S_BRICK model was especially good at predicting the shear-stiffness degradation from very small strain (0.001%) up to the point of failure. The S_BRICK model somewhat overpredicted the amount of dilation, but its prediction was still reasonably good.

Similar results as for sample TT6 were obtained for sample TT7 (Figure 9, see next page), where the strength, stiffness and volumetric response were also significantly better predicted with the S_BRICK model. For sample TT7 it was not possible to compare the stiffness response of the model from very small strains, like for sample TT6, due to measurement difficulties, but based on the available data the S_BRICK model still accurately predicted the stiffness degradation from small strains (0.01%) up to the point of failure.

CONCLUSIONS

Hard soils and soft rocks (HSSR) represent an important part of all geological materials and are often encountered in geotechnical projects around the world. The most convenient definition for HSSR is based on their strength, as proposed by several authors [1], [2], [3], [4], to name just a small selection. A classification that seems to be the most appropriate was proposed by Hawkins and Pinches [5]; this is because it acknowledges the continuum between soils and rocks.

However, from a constitutive modeling standpoint, it is more important to define the theoretical framework through which HSSR can be modeled. It was shown that
the stress-strain behavior of most HSSR can be successfully described using the framework of critical state soil mechanics, as long as the materials are not influenced by discontinuities, and the effects of an excess pore pressure are important [12].

It has also been shown that the higher strength and stiffness encountered in HSSR in comparison to soft soils can be attributed to structure, which represents a key additional parameter in modeling. A very important element of structure is its stability under different stress paths, which also has to be taken into account.

A model for HSSR that includes structure and destructurization named S_BRICK was developed [22], [23] based on the BRICK model [24], [25]. This S_BRICK model was briefly explained here. The comparison of both models’ predictions, the S_BRICK and the BRICK, was carried out on stiff North Sea clay. The predictions of the S_BRICK model were significantly better than the predictions of the BRICK model.

This comparison of the results highlighted the importance of structure and destructurization as key parameters in constitutive models as well as validating the proposition that HSSR can be successfully modeled using the framework of critical state soil mechanics.

The S_BRICK model will have to be further validated on a wider range of materials from soft soils to soft rocks in order to fully confirm its capabilities before it can be incorporated into a numerical environment and used in real boundary-value geotechnical problems.

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