SOIL-NAIL WALL STABILITY ANALYSIS USING ANFIS

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abstract

We present the safety-factor optimization for a soil-nail wall. The optimization is performed using the non-linear programming (NLP) approach. For this purpose, the NLP optimization model OPTINC was developed. The safety factor and the optimal inclination of the soil nails from the horizontal direction depend on the design of the soil-nail wall. Based on these results the ANFIS-INC model was developed for the prediction of the optimal inclination of the soil nail for any design of soil-nail wall. Additionally, an ANFIS-SF model was developed to predict the safety factor for different inclinations of the wall, the slope angle of the terrain, the length of the nails, and the hole diameter. It was found that the inclination of the soil nail should be adjusted to the inclination of wall, the length of nail, the slope angle of the terrain and the hole diameter. With increasing inclination of the wall, the length of the soil nail and the hole diameter, the safety factor is increasing. On the other hand, the safety factor is decreasing with the increasing slope angle of the terrain. The use of nonlinear programming and an Adaptive Network Based Fuzzy Inference System allows a comprehensive analysis of the geotechnical problems.

INTRODUCTION

The idea of combining passive steel reinforcement and shotcrete was first used as a support system for underground excavations in rock. This system is known as the New Austrian Tunneling Method [1,2,3]. The soil-nailing support technique relies on the mobilization of the tensile strength of the steel reinforcement at relatively small deformations in the surrounding soil. The soil-nailing system was first used in 1972 to support an 18-m-high cut-slope near Versailles [4]. Since then, soil nailing was used in many locations in France [5,6], Germany [7,8,9] North America [10] and other countries. The use of soil-nail walls has substantially increased, because it has been demonstrated that soil-nail walls are technically feasible and, in many cases, a cost-effective alternative to the conventional retaining walls used in top-to-bottom excavations in temporary and permanent applications [11].

This paper presents the design analysis of a soil-nail wall. The internal stability of the soil-nail wall was calculated using a plane slip surface. Based on the analytical solutions for the nailed slopes an ANFIS-SF model was built. The depth of the excavation, the material properties of the soil, the number of nails, and the vertical and horizontal spacing of nails are constants, while the inclination of the wall, the slope angle of terrain, the length of the nails and the hole diameter are variables. The ANFIS-SF model predicts a safety factor for the initial design of the soil-nail wall. The safety factor is calculated for each angle (from 1° to 89°) of the slip surface. The critical angle of the slip surface is the one that has the smallest safety factor and depends on the inclination of the soil nails. Non-linear programming (NLP) was used to calculate the optimal inclination of the nails from the horizontal direction for different inclinations of the wall, the slope angle of the terrain, the length of the nails and the hole diameter. Based on the results obtained with NLP an ANFIS-INC model was built that allows a calculation of the optimal inclination of the soil nails for the initial design of the soil-nail wall.
Adaptive network fuzzy inference systems (ANFIS) have been applied to many geotechnical engineering problems and have demonstrated some degree of success [12,13,14, 15,16,17]. For braced excavations in soft clay, Goh et al. [18] developed a neural network model to provide initial estimates of the maximum wall deflections. With an efficient mathematical technique two ANFIS models were developed to predict the safety factor (ANFIS-SF) and the optimal soil-nail inclination (ANFIS-INC).

2 SOIL NAILING

Soil nailing is a construction technique that has many advantages over conventional retaining walls used in top-to-bottom excavations. However, passive anchors are mainly used for the temporary protection of the excavation, and where the displacement of retaining structures are permitted. The soil-nailing technique is carried out in stages:

1. excavation,
2. installation of the nails,
3. installation of the steel reinforcing bars or mesh and drainage,
4. shotcreting,
5. installation of the bearing plates.

These stages are repeated until the bottom of the excavation. The typical spacing between the soil nails is 1‒2 m in the vertical and horizontal directions. The inclination of the soil nails is 5–30° from the horizontal direction. The holes are either cased or uncased depending on the type of soil and are, on average, 50 to 150 mm in diameter [11,19].

Soil nailing is a cost-effective alternative to pre-stressed anchors [20]. Soil nails are cheaper, mainly because they are made from cheaper materials. The use of soil nails is not suitable when the soil properties are inappropriate and displacements of supporting wall are not permitted [21]. Suitable soil properties for the use of passive anchors are:

1. natural cohesive materials, such as silts and low plasticity clays not prone to creep,
2. glacial till,
3. cemented sand with little gravel,
4. fine-to-medium sand with silt to acts as a binder.

Passive anchors are not recommended for the following soil conditions:

1. materials without cohesion,
2. loose granular soils,
3. silt and clay of high plasticity,
4. soft, cohesive soils that will not provide a high pullout resistance,
5. expansive clays,
6. the presence of groundwater.

The corrosion protection in aggressive soils has to be examined very closely in the soil-nailing system. The standard EN 14490:2010 (Execution of special geotechnical works - Soil nailing) establishes general principles for the execution, testing, supervision and monitoring of soil nailing. In aggressive soils, fully encapsulated nails are recommended.

The ultimate limit state (ULS) and serviceability limit state (SLS) are considered in the design and analysis of soil-nail walls. Three ULSs must be verified:

1. the external failure mode,
2. the internal failure mode,
3. the facing failure mode.

The major SLS is excessive wall deformation. According to Byrne [10], the external failure modes are those were failure surface do not intersect the nails and the internal failure modes intersect the nails. The external failure modes are:

1. global stability failure,
2. sliding stability failure,
3. bearing failure.

The internal failure modes are:

4. nail-soil pullout failure,
5. bar-grout pullout failure,
6. nail tensile failure,
7. nail bending failure.

The facing failure modes are:

8. facing flexure failure,
9. facing punching shear failure,
10. headed-stud failure.

Soil-nailed walls can withstand large deformation in all directions; therefore, they perform well during an earthquake. When large wall deformations are not permitted, soil nailing is not an appropriate retaining wall system.
3. PLANE SLIP SURFACE MODEL

An internal stability of soil nail wall could be calculated with different types of slip surface, such as plane slip surface, broken slip surface, parabolic slip surface, circular slip surface and logarithmic spiral slip surface. In this paper, the plane slip surface was used to calculate the safety factor. The geometry and geo-mechanics of the model are presented. The model is designed so that it is possible to change the geometric and geo-mechanical parameters. After determining the geometric design, the safety factor is calculated for each angle (from 1° to 89°) of the slip surface \( \vartheta \). The critical angle of the slip surface is the one that has the smallest safety factor.

3.1 GEOMETRY OF THE SOIL-NAIL WALL

The geometry of the soil-nail wall is shown in Fig. 1. In this model the depth of the excavation wall \((H)\) and the spacings of the soil nails in the vertical direction \((s_v)\) and the horizontal direction \((s_h)\) were determined. The typical spacing between the anchors is 1–2 m and this is the same in the horizontal direction and the vertical direction. Based on these data it is possible to determine the number of anchors \((N_0)\).

The geometry is designed so that it is possible to change the inclination of the wall \((\alpha)\), the slope angle of the terrain \((\beta)\), the length of the nails \((l)\), the inclination of the soil nails \((\eta)\), the spacing of the soil nails in the vertical direction \((s_v)\) and the horizontal direction \((s_h)\).

3.2 GEO-MECHANICAL MODEL OF SOIL-NAIL WALL

In the soil-nailing system for each nail, three bearing capacities are calculated:

1. tensile strength of the nail,
2. pull-out nail-bearing capacity,
3. nail-cap bearing capacity.

The strength characteristics of a nail represent the basic parameters to calculate the actual force in a nail. The tensile strength \([11]\) of the nail is calculated with Eq. (1):

\[
R_{tt} = \frac{\pi \cdot d_n^2 \cdot f_y}{4 \cdot SF_T} 
\]

where:

- \(R_{tt}\) is the strength against breaking (kN),
- \(d_n\) is the nail diameter (mm),
- \(f_y\) is the strength of the nail material (MPa),
- \(SF_T\) is the safety factor against breaking.

The pull-out resistance \([11]\) is calculated with Eq. (2):

\[
T_{tp} = \frac{\pi \cdot d \cdot q_s}{SF_P} 
\]
where:

\( T_{1p} \) is the pull-out nail bearing capacity (kN/m),
\( d \) is the hole diameter (mm),
\( q_{s} \) is the ultimate bond strength (kPa),
\( SF_{p} \) is the safety factor against pull-out.

The nail head strength [11] is calculated with Eq. (3):

\[
R_{1f} = \text{Min} \{ R_{1f}, T_{1p}, l \} \left( \left( 0.6 + 0.2 \cdot s_{\text{max}} \right) - 1 \right)
\]  

(3)

where:

\( R_{1f} \) is the nail-cap bearing capacity (kN),
\( l \) is the length of the soil nail (m),
\( s_{\text{max}} \) is the spacing of the soil nails in the horizontal or vertical direction, whichever is greater (m).

In plane slip a specific slip surface is examined for a variation of the angle \( \vartheta \). In an optimization analysis the calculation is carried out for different angles of the slip surface (From 1° to 89°). The safety factor is calculated for each angle of the slip surface. The ratio of resisting and shear (driving) forces acting on a slip surface should be greater than the minimum safety factor.

The forces acting on a slip surface are:

1. gravitational force parallel to the slip surface,
2. active earth pressure acting on the vertical part of the structure and parallel to the slip surface.

The resisting forces are:

3. the soil friction and cohesion along the slip surface,
4. the sum of the forces transmitted by the nails.

The nail force is determined based on the location of its intersection with the slip surface (see Fig. 1). If a nail is completely in front of the slip surface, then it does not enter the calculation. If a nail crosses the slip surface, then its force is determined with Eq. (4):

\[
F = \text{Min} \{ T_{1p} \cdot x, R_{1f}, R_{1f} + T_{1p} \cdot y \}
\]  

(4)

where:

\( x \) is the nail length behind the slip surface in the direction of the soil body (m),
\( y \) is the nail length in front of the slip surface (m),
\( R_{1f} \) is the strength against breaking (kN),
\( R_{1f} \) is the strength against bearing (kN),
\( T_{1p} \) is the pull-out nail-bearing capacity (kN/m).

For the design, the tensile force distribution along the nail can be simplified, as shown in Fig. 2. The tensile force in the nail increases at a constant slope \( T_{1p} \) (equal to the pullout capacity per unit length), reaches a maximum value, \( R_{1f} \), and then decreases at the rate \( T_{1p} \) to the value \( R_{1f} \) at the nail head.

\[ \text{Figure 2. Distribution of the tensile force along the nail.} \]

The safety factor [11] against global failure \( SF \) is expressed as the ratio of the resisting and driving forces, which act at a tangent to the potential failure plane:

\[
SF = \frac{F_{h} \cdot \cos(\vartheta + \eta) + F_{cd}}{\left( G + S_{a,v} \right) \cdot \sin(\vartheta) + S_{a,v} \cdot \cos(\vartheta)}
\]  

(5)

\[
F_{h} = \sum F_{h,n}
\]  

(6)

\[
F_{cd} = \sum \frac{d_{i}}{d} \left( G \cdot \cos(\vartheta) + F_{h} \cdot \sin(\vartheta + \eta) \right) \cdot \tan(\varphi_{i}) + \sum d_{i} \cdot c_{i}
\]  

(7)

where:

\( G \) is the gravitational force (kN/m),
\( S_{a,v} \) is the vertical component of active pressure (kN/m),
\( S_{a,v} \) is the horizontal component of active pressure (kN/m),
\( d_{i} \) is the length of the \( i \)th section slip surface (m),
\( d_{s} \) is the length of the slip surface (m),
\( F_{h,n} \) is the bearing capacity of the \( n \)th nail behind slip surface (kN/m),
\( c_{i} \) is the cohesion of the \( i \)th soil layer (kPa),
\( \varphi_{i} \) is the angle of the internal friction of the \( i \)th layer (°),
\( \vartheta \) is the inclination of the slip surface (°),
\( \eta \) is the inclination of the nails from the horizontal direction (°).
The active earth pressure (the Coulomb theory) is given by Eq. (8)

$$\sigma_a = \sigma_z \cdot K_a - 2c_f \cdot K_{ac}$$  \hspace{1cm} (8)

where:

- $\sigma_z$ is the vertical geostatic stress (kPa),
- $K_a$ is the coefficient of active earth pressure (-),
- $K_{ac}$ is the coefficient of active earth pressure due to cohesion (-).

The horizontal and vertical components of the active earth pressure are:

$$S_{a,vod} = \sigma_a \cdot \cos(\alpha + \delta) \cdot \frac{H}{2}$$ \hspace{1cm} (9)

$$S_{a,sv} = \sigma_a \cdot \sin(\alpha + \delta) \cdot \frac{H}{2}$$ \hspace{1cm} (10)

where:

- $\delta$ is the angle of the friction structure – soil (°).

### 3.3 OPTIMIZATION MODEL OPTINC

As the optimization problem of the soil-nail wall is non-linear, e.g., the objective function and the (in)equality constraints are non-linear, the non-linear programming (NLP) optimization approach is used and described in the paper. The retaining wall optimization using the NLP approach was presented by several authors [22, 23]. The general NLP optimization problem can be formulated as follows:

$$\text{Max } z = f(x)$$

subjected to:

$$h(x) = 0$$

$$g(x) \leq 0$$

$$x \in X = \left\{ x \in R^n, x^{Lo} \leq x \leq x^{Up}\right\}$$

where $x$ is a vector of the continuous variables defined within the compact set $X$. The functions $f(x)$, $h(x)$ and $g(x)$ are nonlinear functions involved in the objective function $z$, equality and inequality constraints, respectively. All the functions $f(x)$, $h(x)$ and $g(x)$ must be continuous and differentiable. In the context of structural optimization, the variables include dimensions, cross-section characteristics, strains, materials, stresses, etc. The equality and inequality constraints and the bounds of the variables represent a rigorous system of the design, loading, stress and resistance functions taken from the structural analysis and dimensioning.

The optimization of the structures may include various objectives worthy of consideration. In this paper, a geo-mechanical objective function is proposed to maximize the safety factor of the soil-nail wall.

### 3.3.1 Input data

Input data represent the design data (constants) for the optimization. The design data (constants) comprise the height of the soil-nail wall $H$ (m), the vertical $s_v$ (m) and the horizontal $s_h$ (m) spacing of the nails, the number of nails $N_o$ (-), the inclination of the wall $\alpha$ (°), the slope angle of the terrain $\beta$ (°), the length of the nails $l$ (m) and the hole diameter $d$ (mm). In addition, the geo-mechanical data comprise the soil properties, such as the unit weight $\gamma$ (kN/m³), the angle of the internal friction $\varphi_{ef}$ (°), the cohesion of the soil (kPa), the ultimate bond strength of the soil nails in the soil $q_u$ (kPa) and the structure-soil angle of friction $\delta$ (°).

### 3.3.2 Variable

The inclination of the soil nails $\eta$ (°) is declared as a variable in this optimization model (see Fig. 1).

### 3.3.3 Geo-mechanical objective function

The objective function is defined with equation Eq. (11):

$$SF = \text{Max } \left( SF_{crit} \right)$$ \hspace{1cm} (11)

The objective function $SF$ includes the gravitational force $G$ (kN/m), the vertical component of the active pressure $S_{a,sv}$ (kN/m), the horizontal component of the active pressure $S_{a,vod}$ (kN/m), the bearing capacity of the nails behind the slip surface $F_h$ (kN/m), the resisting forces acting on a slip surface $F_{cd}$ (kN/m).

### 3.3.4 Geo-mechanical equality constraint

The critical angle of the slip surface $\varphi_{crit}$ (°) is the one that has the smallest safety factor and depends on the inclination of the soil nails.

$$SF_{crit} = \text{Min } \left( \frac{F_h \cdot \cos(\vartheta + \eta) + F_{cd}}{(G + S_{a,sv}) \cdot \sin(\vartheta) + S_{a,vod} \cdot \cos(\vartheta)} \right)$$ \hspace{1cm} (12)

### 3.3.5 Design (in)equality constraint

The design (in)equality constraint determines the inclination of the soil nails from the horizontal direction $\eta$ (°) to be calculated inside the defined limits.

$$\eta^{Lo} \leq \eta \leq \eta^{Up}$$ (°) \hspace{1cm} (13)
3.3.6 Result of the optimization model
In order to interpret the proposed optimization approach, the paper presents a numerical example. The soil properties of silty sand are given in Table 1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit weight γ</td>
<td>18 kN/m³</td>
</tr>
<tr>
<td>Angle of internal friction φeff</td>
<td>30°</td>
</tr>
<tr>
<td>Cohesion of soil c_eff</td>
<td>5 kPa</td>
</tr>
<tr>
<td>Angle of friction structure - soil δ</td>
<td>20°</td>
</tr>
<tr>
<td>Ultimate bond strength q_s</td>
<td>100 kPa</td>
</tr>
</tbody>
</table>

Table 1. Soil properties of silty sand.

The design data (constants), the initial values, and the lower and upper bounds of the design variable are given in Table 2.

<table>
<thead>
<tr>
<th>Property</th>
<th>Lower bound</th>
<th>Initial value</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of wall H</td>
<td>8 m</td>
<td>8 m</td>
<td>8 m</td>
</tr>
<tr>
<td>Vertical spacing s_v</td>
<td>1.5 m</td>
<td>1.5 m</td>
<td>1.5 m</td>
</tr>
<tr>
<td>Horizontal spacing s_h</td>
<td>1.5 m</td>
<td>1.5 m</td>
<td>1.5 m</td>
</tr>
<tr>
<td>Inclination of wall α</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>Slope angle of terrain β</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>Length of nail l</td>
<td>6 m</td>
<td>6 m</td>
<td>6 m</td>
</tr>
<tr>
<td>Hole diameter d</td>
<td>100 mm</td>
<td>100 mm</td>
<td>100 mm</td>
</tr>
</tbody>
</table>

Table 2. Design data, initial values, lower and upper bounds of the design variable (see Fig. 1).

The value of the safety factor is changing with the inclination of the nails η. The optimal inclination of the soil nails from the horizontal direction η_{opt} is defined with the maximum safety factor (see Fig. 3). In this numerical example the optimal inclination of the soil nails from the horizontal direction is η_{opt} = 19.1°.

4 ADAPTIVE-NETWORK-BASED FUZZY INFERENCE SYSTEM

The basic structure of the fuzzy inference system (FIS) was introduced by Zadeh [24]. In this type of FIS it is essential to predetermine the rule structure and the membership functions. Human-determined membership functions are subjective and are different from person to person. Standard methods that transform human knowledge or experience into fuzzy rules and membership functions do not exist. Usually, there is a collection of input/output data, which we would like to use to construct the FIS model. The effective method for tuning the membership functions and minimizing the output error measure is the Adaptive-Network-Based Fuzzy Inference System (ANFIS). The ANFIS [25] uses given input/output data to construct a FIS, whose membership function parameters are tuned (adjusted) using either a back-propagation algorithm alone or in combination with a least-squares type of method. This adjustment allows fuzzy systems to learn from the data they are modelling. ANFIS only supports Sugeno-Takagi-Kang [26] identification models, which should

<table>
<thead>
<tr>
<th>Property</th>
<th>ANFIS-SF</th>
<th>ANFIS-INC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of wall H (m)</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Vertical spacing s_v (m)</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Horizontal spacing s_h (m)</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>s_{sup} (m)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>s_{bottom} (m)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Number of nails No (-)</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Inclination of soil nail η (°)</td>
<td>10</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3. Input and output data for two ANFIS models.

<table>
<thead>
<tr>
<th>Property</th>
<th>ANFIS-SF</th>
<th>ANFIS-INC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>ANFIS-SF</td>
<td>ANFIS-INC</td>
</tr>
<tr>
<td>Height of wall H (m)</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Vertical spacing s_v (m)</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Horizontal spacing s_h (m)</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>s_{sup} (m)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>s_{bottom} (m)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Number of nails No (-)</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Inclination of soil nail η (°)</td>
<td>10</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3. Input and output data for two ANFIS models.
have only one output parameter. The adaptive network is a superset of all kinds of feedforward neural networks with a supervised learning capability [27]. ANFIS is a fuzzy inference system implemented in the framework of adaptive networks and uses the advantages of neural networks and fuzzy logic.

The safety factor $SF$ and the optimal inclination of the soil nails from the horizontal direction $\eta_{opt}$ depend on the design of the soil-nail wall. Therefore, the optimal inclination was calculated with the optimization model OPTINC for a different inclination of the wall $\alpha$ (°), the slope angle of terrain $\beta$ (°), the length of the nails $l$ (m) and the hole diameter $d$ (mm). Based on these results the ANFIS-INC model was developed for the prediction of the optimal inclination of the soil nail for any design of soil-nail wall. Additionally, the ANFIS-SF model was developed to predict the safety factor for different inclinations of the wall $\alpha$ (°), the slope angle of terrain $\beta$ (°), the length of the nails $l$ (m) and the hole diameter $d$ (mm).

### 4.1 Results of Geo-Mechanical and Optimization Model

The soil properties of the geo-mechanical model are given for silty sand (see Table 1). The design data (constants) are given in Table 3. The series of calculations for the safety factor $SF$ (-) and the optimal inclination soil nails $\eta$ (°) are given in Table 4.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$ (m)</td>
<td>$\alpha$ (°)</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
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<tr>
<td>4</td>
<td>0</td>
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<td>6</td>
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<td>8</td>
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<td>20</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
</tr>
</tbody>
</table>
shows that the calculations were made for 81 different combinations of design data.

4.2 ANFIS STRUCTURE

For a Sugeno fuzzy model ANFIS-SF, a rule set with $n$ fuzzy “If-then” is as follows:

Rule 1: If $l$ is In1MF1 and $\alpha$ is In2MF1 and $\beta$ is In3MF1 and $d$ is In4MF1 then

$$S_{F1} = k_{11} \cdot l + k_{12} \cdot \alpha + k_{13} \cdot \beta + k_{14} \cdot d + k_{15}$$  (14)

Rule 2: If $l$ is In1MF2 and $\alpha$ is In2MF2 and $\beta$ is In3MF2 and $d$ is In4MF2 then

$$S_{F2} = k_{21} \cdot l + k_{22} \cdot \alpha + k_{23} \cdot \beta + k_{24} \cdot d + k_{25}$$  (15)

where $k_{11}, k_{12}, k_{13}, k_{14}, k_{15}, k_{21}, k_{22}, k_{23}, k_{24}, k_{25}$ are the consequent parameters and $l, \alpha, \beta$ and $d$ are the input variables. The sign In1MF1 stands for the membership function 1 in input 1. The output of each rule is equal to a constant and the final output is the weighted average of each rule’s output.

$$SF = \sum_{i=1}^{n} w_i \cdot S_{Fi}$$  (16)

$$\eta_{opt} = \sum_{i=1}^{n} w_i \cdot \eta_{opt,i}$$  (17)

The weights are obtained from the Gaussian membership function.

$$\mu_A(x) = \exp \left( -\frac{(x-c)^2}{\sqrt{2} \sigma} \right)$$  (18)

where $c$ is the position of the centre of the curve’s peak and $\sigma$ is the width of the curve. The parameters $c$ and $\sigma$ are the premise parameters. The first membership grade of a fuzzy set ($In1MF_1, In2MF_1, In3MF_1, In4MF_1$) is calculated with the following equations:

$$\mu_{In1MF_1}(l) = \exp \left( -\frac{(l-c_{In1MF_1})^2}{\sqrt{2} \sigma_{In1MF_1}} \right)$$  (19)

$$\mu_{In2MF_1}(\alpha) = \exp \left( -\frac{(\alpha-c_{In2MF_1})^2}{\sqrt{2} \sigma_{In2MF_1}} \right)$$  (20)

$$\mu_{In3MF_1}(\beta) = \exp \left[ -\frac{(\beta-c_{In3MF_1})^2}{2 \sigma_{In3MF_1}} \right]$$  (21)

$$\mu_{In4MF_1}(d) = \exp \left[ -\frac{(d-c_{In4MF_1})^2}{2 \sigma_{In4MF_1}} \right]$$  (22)

where $l, \alpha, \beta$ and $d$ are the inputs to the Gaussian membership function. Next, the product of the membership function for every rule is calculated:

$$w_i = \mu_{In1MF_1}(l) \cdot \mu_{In2MF_1}(\alpha) \cdot \mu_{In3MF_1}(\beta) \cdot \mu_{In4MF_1}(d)$$  (23)

where $w_i$ represents the firing strength of the rule $i$. The ratio of the $i$th rule’s firing strength to the sum of all the rules’ firing strengths is defined with:

$$\frac{w_i}{w_1 + w_2 + \ldots + w_n} , \text{ for } i = 1, 2, \ldots, n.$$  (24)

The input data represents a node on the left, and the right node represents the output data (Fig. 4).

In order to achieve the desired input-output mapping, the consequent and premise parameters are updated according to the given training data (Table 4) and the hybrid learning procedure. This hybrid learning procedure [25] is composed of a forward pass and a backward pass. In the forward pass, the algorithm uses the least-squares method to identify the consequent parameters. In the backward pass the errors are propagated backwards and the premise parameters are updated with the gradient descent. The premise ($\sigma_i, c_i$) and the consequent ($k_i$) parameters of the ANFIS-SF model and ANFIS-INC model are given in Table 5. By setting the premise and consequent parameters, the space of four variables is described.

4.3 TESTING THE ANFIS MODELS

To test and validate the ANFIS models, a data set was selected that was not used during the training of the network (Table 6).

The coefficient of determination (R-square) and the root-mean-squared error (RMSE) between the predicted and the calculated values is taken as a measure of the performance. The calculated and predicted safety factors SFs for the model ANFIS-SF are shown in Fig. 5.
Figure 4. Structure of ANFIS-SF and ANFIS-INC model.

Table 5. The premise and consequent parameter for the ANFIS-SF and ANFIS-INC model.

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<th>ln2MF1</th>
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Figure 5. Calculated versus predicted safety factor SF.
The comparison between the optimal inclination of the soil nail calculated with the optimization model OPTINC and the ANFIS-INC model is shown in Fig. 6.

The higher coefficient of determination means that the correlation between the monitored and predicted data is high; it does not mean that the monitored data are close to the predicted data. To estimate the error of each model we used RMSE.

## DISCUSSION

The presented ANFIS models are able to predict the safety factor \( SF (-) \) and the optimal inclination of the soil nails \( \eta (°) \) for different inclinations of the wall \( \alpha (°) \), the slope angle of the terrain \( \beta (°) \), the length of the nails \( l (m) \) and the hole diameter \( d (mm) \). Fig. 7 shows the safety factor depending on the length of the soil nails and the inclination of the wall. With increasing inclination of

### Table 6. Testing data compared to ANFIS models.

| \( l \) (m) | \( \alpha \) (°) | \( \beta \) (°) | \( d \) (mm) | \( SF \) (-) | ANFIS-SF (-) | Error (%) | OPTINC (°) | ANFIS-INC (°) | Error (%) |
|------|----------|----------|--------|---------|-----------|---------|---------|---------|---------|---------|
| 5    | 5        | 10       | 75     | 0.91    | 0.89      | 2.1     | 11.1    | 10.2    | 8.1     |
| 7    | 5        | 10       | 75     | 1.15    | 1.14      | 1.2     | 12.0    | 12.3    | 2.8     |
| 5    | 5        | 10       | 125    | 1.08    | 1.08      | 0.3     | 16.2    | 17.8    | 10.0    |
| 7    | 5        | 10       | 125    | 1.38    | 1.41      | 2.3     | 19.1    | 20.6    | 7.9     |
| 5    | 15       | 10       | 75     | 1.09    | 1.07      | 1.9     | 17.3    | 15.2    | 12.1    |
| 7    | 15       | 10       | 75     | 1.34    | 1.33      | 1.0     | 17.0    | 16.8    | 1.1     |
| 5    | 15       | 10       | 125    | 1.27    | 1.27      | 0.3     | 21.7    | 22.7    | 4.6     |
| 7    | 15       | 10       | 125    | 1.58    | 1.61      | 1.8     | 23.7    | 24.6    | 3.7     |
| 5    | 5        | 20       | 75     | 0.84    | 0.81      | 3.0     | 10.1    | 8.9     | 11.5    |
| 7    | 5        | 20       | 75     | 1.04    | 1.02      | 1.9     | 12.0    | 11.5    | 4.0     |
| 5    | 5        | 20       | 125    | 0.98    | 0.97      | 0.7     | 15.5    | 16.6    | 6.9     |
| 7    | 5        | 20       | 125    | 1.22    | 1.24      | 1.4     | 18.7    | 20.3    | 8.2     |
| 5    | 15       | 20       | 75     | 1.01    | 0.98      | 2.7     | 14.5    | 14.0    | 3.4     |
| 7    | 15       | 20       | 75     | 1.21    | 1.19      | 1.7     | 16.9    | 16.1    | 4.6     |
| 5    | 15       | 20       | 125    | 1.15    | 1.15      | 0.4     | 23.4    | 21.6    | 8.0     |
| 7    | 15       | 20       | 125    | 1.38    | 1.40      | 1.3     | 23.9    | 24.4    | 1.8     |

Figure 6. Calculated versus predicted optimal inclination of soil nails \( \eta \).
the wall, the length of the soil nail and the hole diameter, the safety factor is increasing. On the other hand, the safety factor is decreasing with the increasing slope angle of the terrain. The larger hole diameter increases the pull-out resistance; therefore, the safety factor is larger.

Fig. 8 shows the optimal inclination of the soil nails depending on the length of the soil nails and the inclination of the wall. With increasing inclination of the wall, the optimal inclination of the soil nails increases. The lengths of the soil nails have a small influence on the optimal inclination of the soil nail in comparison with the inclination of the wall.

6 CONCLUSIONS

The paper presents the safety-factor optimization for a soil-nail wall. The optimization is performed using the non-linear programming (NLP) approach. For this purpose, the NLP optimization model OPTINC was developed. The model comprises the safety-factor objective function, which is subjected to geo-mechanical and design constraints. As the model was developed in a general form, the optimization of the system can be performed for different heights of the wall, spacings of the nails as well as for different soil environments. The output of the OPTINC model is an optimal inclination of the soil nails.

The safety factor and the optimal inclination of the soil nails from the horizontal direction depends on the design of the soil-nail wall. Therefore, the safety factor $SF$ and the optimal inclination of the nails $\eta$ were calculated for different inclinations of the wall $\alpha$, the slope angle of the terrain $\beta$, the length of the nails $l$ (m) and the hole diameter $d$ (mm). Based on these results two ANFIS models were developed. The ANFIS-
SF model is used to predict the SF factor and the ANFIS-INC model is used to predict the optimal inclination of the nails for different design data.

In the present study only a plane slip surface is used to calculate the internal stability, while other types of slip surface, such as a broken slip surface, parabolic slip surface, circular slip surface and logarithmic spiral slip surface, should also be calculated. It was found that the inclination of soil nail should be adjusted to the inclination of the wall, the length of the nail, the slope angle of the terrain and the hole diameter.

REFERENCES


