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A computational model for determination of service life of gears

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Abstract

A computational model for determination of service life of gears in regard to bending fatigue in a gear tooth root is presented. The fatigue process leading to tooth breakage is divided into crack initiation and crack propagation period. The strain-life method in the framework of the FEM-method has been used to determine the number of stress cycles N_i required for the fatigue crack initiation, where it is assumed that the crack is initiated at the point of the largest stresses in a gear tooth root. The simple Paris equation is then used for the further simulation of the fatigue crack growth. The functional relationship between the stress intensity factor and crack length $K = f(a)$, which is needed for determination of the required number of loading cycles N_p for a crack propagation from the initial to the critical length, is obtained using displacement correlation method in the framework of the FEM-method. The total number of stress cycles N for the final failure to occur is then a sum of N_i and N_p. The model is used for determination of service life of real spur gear made from through-hardened steel 42CrMo4, where required material parameters have been determined previously by the appropriate test specimens.

Keywords: Machine elements; Fatigue initiation; Fatigue crack growth; Numerical modelling

1. Introduction

Two kinds of teeth damage can occur on gears under repeated loading due to fatigue; namely the pitting of gear teeth flanks and tooth breakage in the tooth root [1]. In this paper only the tooth breakage is addressed and the developed computational model is used for calculation of tooth bending strength, i.e. the service life of gear tooth root.

Several classical standardised procedures (DIN, AGMA, ISO, etc.) can be used for the approximate determination of load capacity of gear tooth root. They are commonly based on the comparison of the maximum tooth-root stress with the permissible bending stress [1]. Their determination depends on a number of different coefficients that allow for proper consideration of real working conditions (additional internal and external dynamic forces, contact area of engaging gears, gear's material, surface roughness, etc.). The classical procedures are exclusively based on the experimental testing of the reference gears and they consider only the final stage of the fatigue process in the gear tooth root, i.e. the occurrence of final failure.

However, the complete process of fatigue failure of mechanical elements may be divided into the following stages [2–5]: (1) microcrack nucleation; (2) short crack growth; (3) long crack growth; and (4) occurrence of final failure. In engineering applications the first two stages are usually termed as "crack initiation period", while long crack growth is termed as "crack propagation period". An exact definition of the transition from initiation to propagation period is usually not possible. However, the crack initiation period generally account for most of the service life, especially in high-cycle fatigue, see Fig. 1. The complete service life of mechanical elements can than be determined from the number of stress cycles N_i required for the fatigue crack initiation and the number of stress cycles N_p required for a crack to propagate from the initial to the critical crack length, when the final failure can be expected to occur:

$$N = N_i + N_p \quad (1)$$

One of the most convenient representations of the fatigue crack growth is the Kitagawa–Takahashi plot of applied stress range required for crack growth, $\Delta\sigma$,

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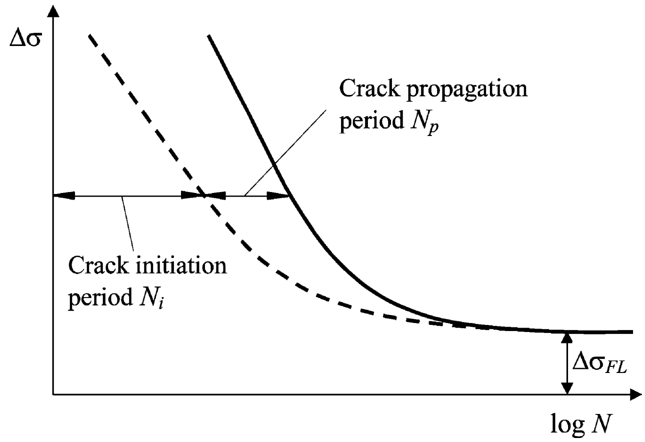

Fig. 1. Schematic representation of the service life of mechanical elements.

against crack length, a, using logarithmic scales, as shown in Fig. 2a [6]. If the relation between the stress intensity range ΔK and the crack length a is used to describe the same diagram, it will be equally drawn as the form in Fig. 2b [7]. In the area of a constant value of threshold stress intensity range ΔK_{th} the linear elastic fracture mechanics (LEFM) can be used to analyse the fatigue crack growth. The threshold crack length a_{th}, below which LEFM is not valid, may be estimated approximately as [7]:

$$a_{th} \approx \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta \sigma_{FL}}\right)^2 \quad (2)$$

where $\Delta\sigma_{FL}$ is the fatigue limit, see Figs. 1 and 2. The threshold crack length a_{th} thus defines the transition point between short and long cracks, i.e. the transition point between initiation and propagation period in engineering applications. However, a wider range of values have been selected for a_{th} in the literature, usually between 0.05 and 1 mm for steels where high strength steels take the smallest values [7,14,15].

Fracture mechanics has developed into a useful discipline for predicting strength and life of cracked gear tooth and many authors have used this theory for calculation of tooth bending strength. The short review of the earlier studies in this field is given in [8] and [9]. The authors have performed a comprehensive study to describe the fatigue crack growth using fracture mechanics theory, where the numerical procedures like Finite Element Method (FEM) and Boundary Element Method (BEM) are usually used for that purpose. In some subsequent studies authors tried to observe the specific effects on the fatigue crack growth. Kato et al. [8] have developed a method to simulate the fatigue crack growth in a carburized gear tooth, where the effect of residual stresses is taken into account. In their model, the crack initiation period was neglected and the entire service life of gear tooth was assumed to be the process of crack propagation. Blarasin et al. [9] have studied the problem of fatigue crack propagation in specimens similar to gear teeth considering the influence of different surface treatment on the service life. Lewicki and Ballarini [10] investigated the effect of gear rim thickness on crack propagation in a gear tooth root. In their study, the crack initiation period was determined experimentally. The effect of crack closure on crack propagation in a gear tooth root was performed by Guagliano and Vergani [11]. The results of their investigations show that the crack closure effect may be significant when the load applied is lower.

It is evident from the about studies that the authors focused mostly on the period of the fatigue crack growth in a gear tooth root, where computational or experimental methods are used. Therefore, the main purpose of this paper is to present a complete computational model for determination of service life of a gear tooth root, where both, crack initiation and crack propagation period are analysed using appropriate numerical models. The proposed model is used on a real spur gear pair made from high strength alloy steel (through-hardened)

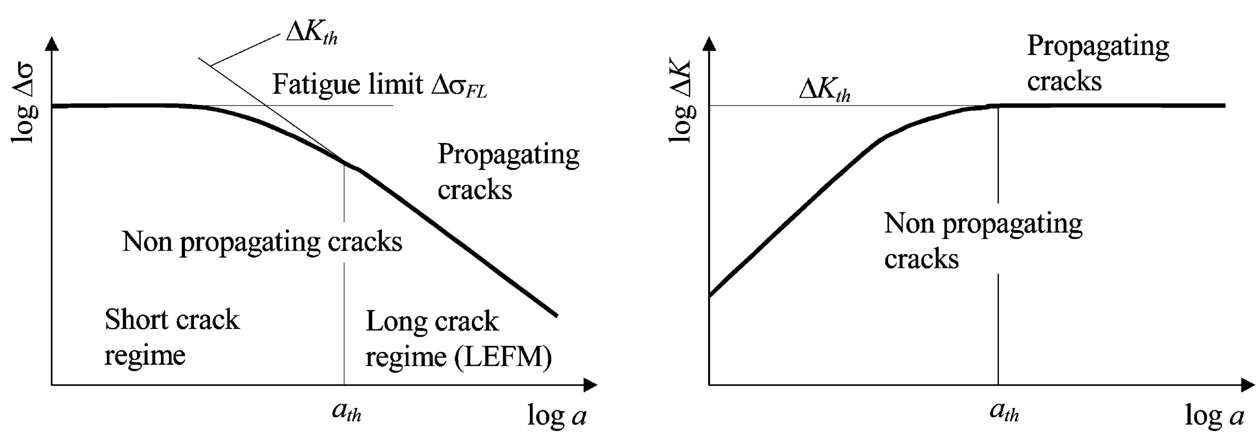

Fig. 2. Variation of the applied stress range with the crack length (a) and variation of the stress intensity range with the crack length (b).

without additional surface treatment (case-hardening or shot-peening). In fact, the residual stresses in surface layer are in that case smaller if compared with the case-hardened gears and are not considered in this study. Furthermore, the material parameters used in the computational analysis have been determined previously by the appropriate test specimens, where the material and thermal treatment (through-hardening) were the same as by treated gears.

2. Fatigue crack initiation

The initiation of fatigue cracks represents one of the most important stages in the fatigue process. Position and mode of fatigue crack initiation depends on the microstructure of a material, the type of the applied stress and micro- and macro-geometry of the specimen. The initiation phase of fatigue life in a virgin material is often assumed to constitute the growth of short cracks up to the size a_{th} (see Fig. 2), which is the transition length of short cracks into long cracks.

The fatigue crack initiation includes the early development of fatigue damage [12] and is strongly dependent on the size scale of observation. For example, materials scientists are likely to consider the nucleation of flaws along persistent slip bands (*PSB*) as the initiation stage of fatigue damage, whilst mechanical engineers may associate the resolution of crack detection with the threshold for crack nucleation. Between this wide range of view-points lies a variety of failure mechanisms that are affiliated with the inception of microscopic flaws at grain boundaries, twin boundaries, inclusions, as well as microscopic and macroscopic stress concentration [13]. It is quite difficult to find full agreement on what is meant by the term short crack but for steels this might be all cracks less than 1 mm [14].

The model for the fatigue crack initiation presented here is based on the continuum mechanics approach, were it is assumed that the material is homogeneous and isotropic, i.e. without imperfections or damages. Methods for the fatigue analyses are in that case usually based on the Coffin–Manson relation between deformations (ε), stresses (σ) and the number of loading cycles (N_i). However, the strain-life method ($\varepsilon - N_i$) is usually used to determine the number of stress cycles N_i required for the fatigue crack initiation, where it is assumed that the crack is initiated at the point of the largest stresses in the material. The total cyclic strain range $\Delta\varepsilon$ comprises two components (elastic and plastic cyclic strain range $\Delta\varepsilon_e$ and $\Delta\varepsilon_p$) and can be described as [12,13,15]:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\Delta\sigma}{2E} + \varepsilon_f^{'}\left(\frac{\Delta\sigma}{2\sigma_f^{'}}\right)^{\frac{1}{n'}} = \frac{\sigma_f^{'}}{E}(2N_i)^b + \varepsilon_f^{'}(2N_i)^c \qquad (3)$$

where $\Delta\sigma$ is the applied stress range, E is the Young's modulus, n' is the cyclic strain hardening exponent, $\sigma_f^{'}$ is the fatigue strength coefficient, $\varepsilon_f^{'}$ is the fatigue ductility coefficient, b is the exponent of strength and c is the fatigue ductility exponent, see Fig. 3. The number of stress cycles N_i required for the fatigue crack initiation can than be solved iterative from Eq. (3) for the applied stress range $\Delta\sigma$ and the appropriate material parameters E, n', $\sigma_f^{'}$, $\varepsilon_f^{'}$, b and c.

It is know from the practical applications that the fatigue failures on gears are usually nucleated at the surface and so surface conditions become an extremely important factor influencing fatigue strength. Normally, scratches, pits, machining marks etc. influence fatigue strength by providing additional stress raisers which aid the process of crack nucleation. Broadly speaking, high strength steels are more adversely affected by a rough surface finish than softer steels. Therefore, the influence of surface finish on fatigue strength is strongly related to tensile strength of the material. The surface finish correction factor C_{sur} is presented in Fig. 4 in dependence on surface roughness R_a and tensile strength of the material R_m [15]. Using this assumption the real service life of gears may be reduced in regard to the appropriate value of C_{sur}, which is, in the present study, considered through the following equation:

$$\Delta\sigma_{FLr} = \Delta\sigma_{FR}\cdot C_{sur} \qquad (4)$$

where $\Delta\sigma_{FLr}$ is the real fatigue limit and $\Delta\sigma_{FL}$ is the fatigue limit of the polished laboratory specimen.

3. Fatigue crack propagation

The application of LEFM to fatigue is based upon the assumption that the fatigue crack growth rate, da/dN, is a function of the stress intensity range $\Delta K = K_{max} - K_{min}$, where a is a crack length and N is a number of

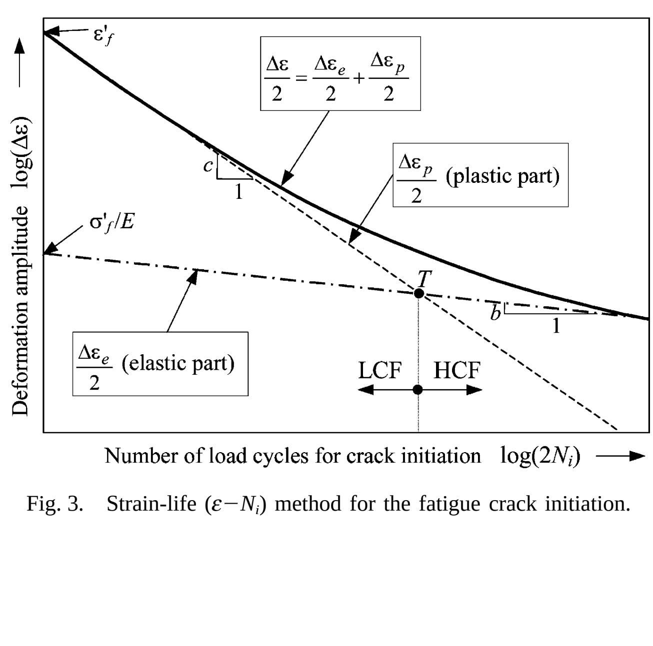

Fig. 3. Strain-life ($\varepsilon - N_i$) method for the fatigue crack initiation.

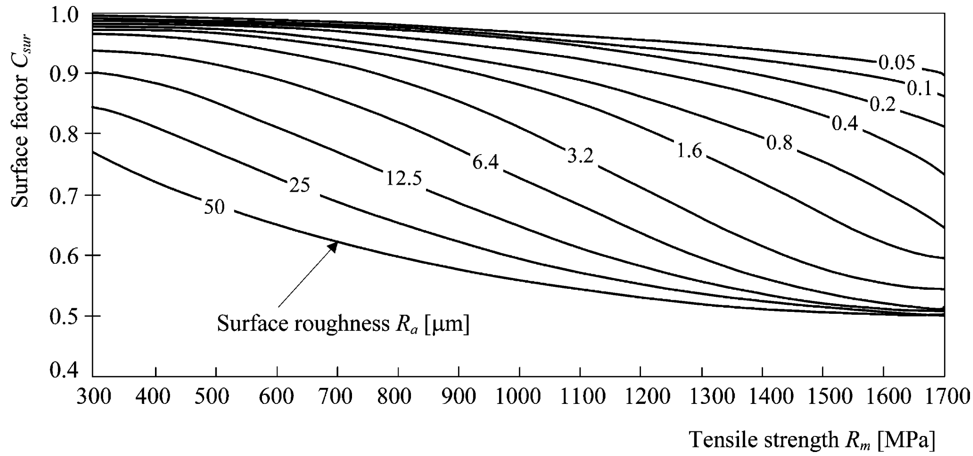

Fig. 4. Surface finish correction factor C_{sur}.

loading cycles. In this study the simple Paris equation is used to describe of the crack growth rate [16]:

$$\frac{da}{dN} = C[\Delta K(a)]^m \tag{5}$$

where C and m are the material parameters. In respect to the crack propagation period N_p according to Eq. (1), and with integration of Eq. (5) one can obtain the number of loading cycles N_p :

$$\int_0^{N_p} dN = \frac{1}{C} \cdot \int_{a_{th}}^{a_c} \frac{da}{[\Delta K(a)]^m} \tag{6}$$

Eq. (6) indicates that the required number of loading cycles N_p for a crack to propagate from the initial length a_{th} to the critical crack length a_c can be explicitly determined, if C, m and $\Delta K(a)$ are known. C and m are material parameters and can be obtained experimentally, usually by means of a three point bending test as to the standard procedure ASTM E 399-80 [17]. For simple cases the dependence between the stress intensity factor and the crack length $K = f(a)$ can be determined using the methodology given in [16,17]. For more complicated geometry and loading cases it is necessary to use alternative methods. In this work the Finite Element Method in the framework of the programme package Franc2d [18] has been used for simulation of the fatigue crack growth. In this approach the determination of the stress intensity factor is based on the displacement correlation method using singular quarter-point, six node triangular elements around the crack tip, Fig. 5. The stress intensity factor in mixed mode plane strain condition can then be determined from the nodal displacements as:

$$K_I = \frac{2G}{(3-4\nu) + 1} \cdot \sqrt{\frac{\pi}{2L}} \cdot [4v_d - v_e - 4v_b + v_c]; \tag{7}$$

$$K_{II} = \frac{2G}{(3-4\nu) + 1} \cdot \sqrt{\frac{\pi}{2L}} \cdot [4u_d - u_e - 4u_b + u_c]$$

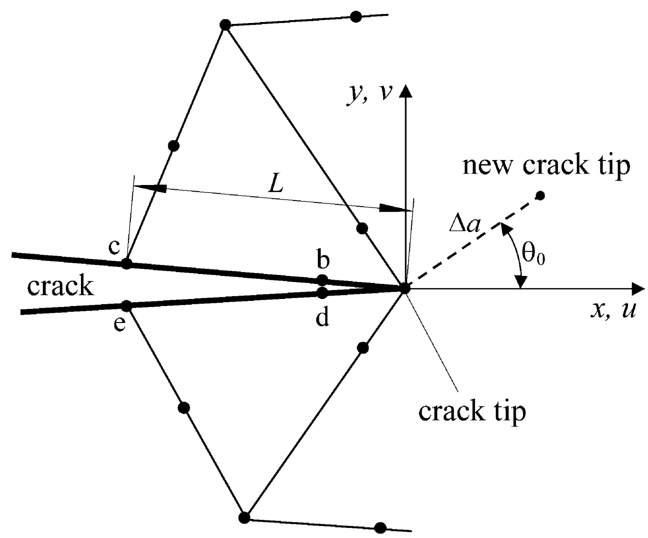

Fig. 5. Triangular quarter-point elements around crack tip.

where G is the shear modulus of the material, ν is the Poisson ratio, L is the finite element length on crack face, u and v are displacements of the finite element nodes b, c, d and e, see Fig. 5. The combined stress intensity factor is then:

$$K = \sqrt{(K_I^2 + K_{II}^2) \cdot (1-\nu^2)} \tag{8}$$

The computational procedure is based on incremental crack extensions, where the size of the crack increment is prescribed in advance. In order to predict the crack extension angle the maximum tangential stress criterion (MTS) is used. In this criterion it is proposed that crack propagates from the crack tip in a radial direction in the plane perpendicular to the direction of greatest tension (maximum tangential tensile stress). The predicted crack propagation angle (see Fig. 5) can be calculated by:

$$\theta_0 = 2\tan^{-1}\left[\frac{1}{4} \cdot \frac{K_I}{K_{II}} \pm \sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8}\right] \tag{9}$$

Table 1
Basic data of a treated spur gear

Profile	Involute
Normal module	m_n = 4.5 mm
Number of teeth	z = 39
Pressure angle on pitch circle	α_n = 24°
Coefficient of profile displacement	x = 0.06
Tooth width	B = 28 mm
Gear material	42CrMo4

A new local remeshing around the new crack tip is then required. The procedure is repeated until the stress intensity factor reaches the critical value K_c, when the complete tooth fracture is expected. Following the above procedure, one can numerically determine the functional relationship $K = f(a)$.

4. Practical example

The presented model has been used for the computational determination of the service life of real spur gear with complete data set given in Table 1. The gear is made of high strength alloy steel 42CrMo4 (0.43 %C, 0.22 %Si, 0.59 %Mn, 1.04 %Cr, 0.17 %Mo) with Young's modulus $E = 2.1 \times 10^5$MPa and Poison's ratio $\nu = 0.3$. The gear material is thermally treated (through-hardening) as follows: flame heated at 810°C; 2 min, hardened in oil; 3 min and tempered at 180°C; 2 h.

4.1. Fatigue crack initiation

The strain-life method ($\varepsilon - N_i$) in the framework of the FEM program package MSC/FATIGUE [15] has been used to determine the number of stress cycles N_i required for the fatigue crack initiation. The material parameters $n' = 0.14$, $\sigma'_f = 1820$Mpa, $\varepsilon'_f = 0.65$, $b = -0.08$ and $c = -0.76$ according to Eq. (3) have been taken from the material database available in [15]. On the basis of gear data set given in Table 1 the finite element model shown in Fig. 6 has been constructed for the further numerical calculation of the stress–strain field in a gear tooth root for the plain strain conditions. The gear tooth was loaded with normal pulsating force F (different values of F have been considered in the numerical computations, see Table 2) which is acting at the outer point of single tooth contact. The computational analyses have been performed at the point where maximum principal stresses occur in a gear tooth root, see Fig. 6. The influence of surface finish on fatigue strength has been considered with the surface finish factor C_{sur} as described in Section 2 for different surface roughness (R_a = 0.8, 3.2 and 6.4 μm). The results of such numerical computations are presented in Table 2.

4.2. Fatigue crack propagation

The FEM-programme package Franc2d described in Section 3 has been used for the numerical simulation of the fatigue crack growth. The initial crack was placed perpendicularly to the surface at the point where the crack initiation has been determined previously, see Fig. 7. In numerical computations it has been assumed that the initial crack corresponds to the threshold crack length a_{th}, see Section 1. Considering the material parameters $\sigma_{FL} \approx 550$MPa and $K_{th} = 269$MPa $\sqrt{\text{mm}}$ [19,20] the threshold crack length is equal to $a_{th} \approx 0.1$mm. The fracture toughness $K_{Ic} \approx 2620$MPa $\sqrt{\text{mm}}$, and the material parameters $C = 3.31 \times 10^{-17}$ mm/cycl/(MPa$\sqrt{\text{mm}}$)^m and $m = 4.16$ have been determined previously by the three-point bending samples according to ASTM E 399-80 standard and for the same material as used in this study [19].

The tooth loading was set equal as by the numerical analysis of the fatigue crack initiation, see Section 4.1. During numerical simulations the crack increment size Δa was 0.2 mm up to the crack length a = 4mm, and after this 0.4 mm up to the critical crack length a_c, see Fig. 5. To be able to determine the number of loading cycles N_p required for the crack to propagate from the initial crack length a_{th} to the critical crack length a_c according to Eq. (6), it is necessary to determine the function $\Delta K = f(a)$ first. Fig. 8 shows the functional

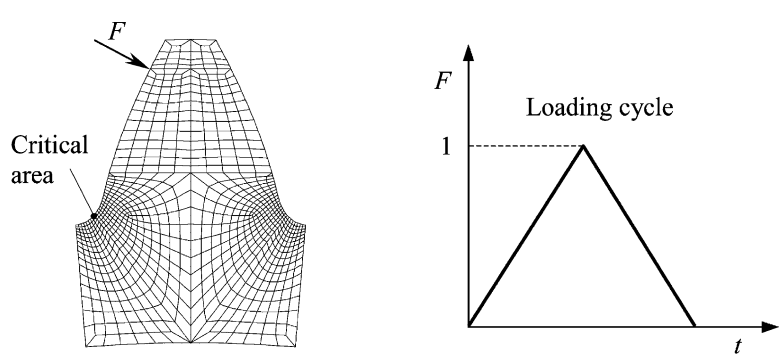

Fig. 6. Finite element model with the pattern of loading cycle for the fatigue analysis.

Table 2
Computational results for the fatigue crack initiation period depending on surface finish of gear teeth flanks

Loading F [N/mm]	Maximum principal stress in a gear tooth root σ [MPa]	Number of stress cycles for the fatigue crack initiation N_i		
		R_a = 6.4 μm	R_a = 3.2 μm	R_a = 0.8 μm
800	527	1.368×10^7	5.270×10^7	2.327×10^8
1000	659	2.327×10^6	6.732×10^6	2.049×10^7
1200	790	4.370×10^5	1.144×10^6	3.141×10^6
1400	922	9.811×10^4	2.292×10^5	5.677×10^5
1600	1050	3.151×10^4	6.504×10^4	1.446×10^5

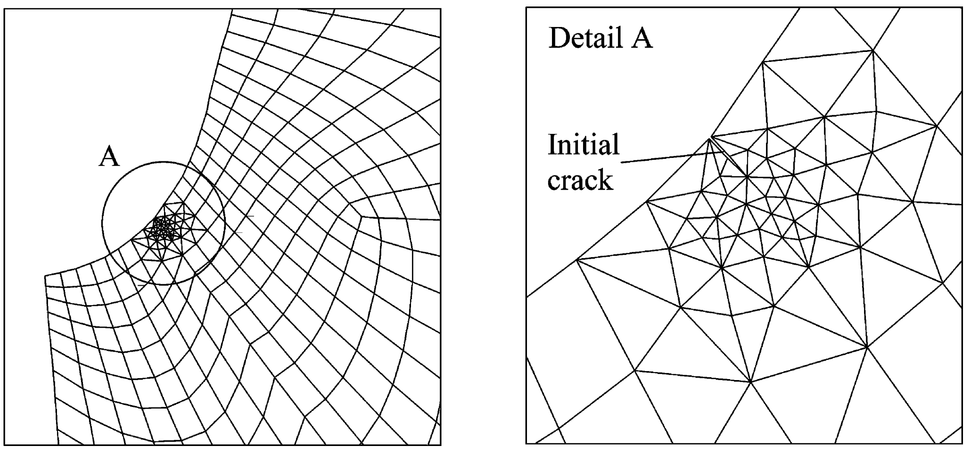

Fig. 7. Finite element mesh around initial crack in a gear tooth root.

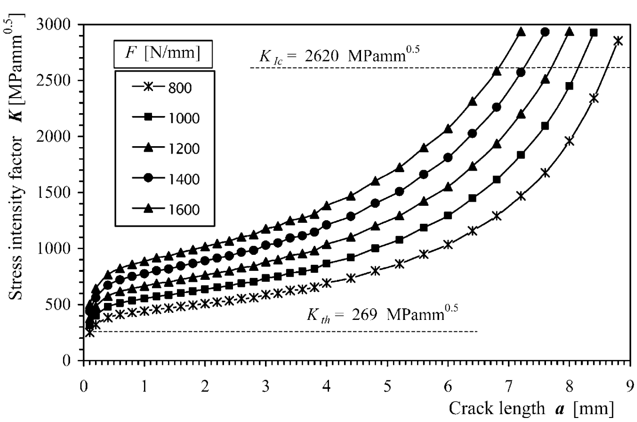

Fig. 8. Functional relationship between the stress intensity factor and crack length.

relationship between the combined stress intensity factor K and crack length a, where K is obtained with Eq. (8) using numerically determined values of K_I and K_{II}. Numerical analyses have shown that the K_I stress intensity factor is much higher if compared with K_{II} (K_{II} was less than 5 % of K_I for all load cases and crack lengths). Therefore, the fracture toughness K_{Ic} can be considered as the critical value of K and the appropriate crack length can be taken as the critical crack length a_c. The loading cycles N_p for the crack propagation to the critical crack length can be estimated using Eq. (6), see Table 3. Fig. 9 shows the numerically determined crack propagation path in a gear tooth root.

On the basis of the computational results for crack initiation (N_i) and crack propagation (N_p) period in Tables 2 and 3 the complete service life of gear tooth root can be obtained according to Eq. (1), see Fig. 10. It is evident from Fig. 10 that the ratio among the periods of initiation and end of propagation (i.e. final breakage) depends on the stress level. At low stress level almost all service life is spent in crack initiation, but at high stress levels the significant part of the life is spent in the crack propagation (note that the abscissa of the diagram has a logarithmic scale). The computational results are compared with the available experimental results [20], which have been obtained using FZG-test machine for the same material and same thermal treatment as used in this study. The comparison of computational and experimental results shows a reasonable agreement, see Fig. 10.

5. Conclusions

The paper presents a computational model for determination of service life of gears in regard to bending fatigue in a gear tooth root. The fatigue process leading

Table 3
Gear tooth crack propagation life of the cracked tooth

Loading F [N/mm]	Critical crack length a_c [mm]	Number of cycles N_p
800	8.6	1.160×10^6
1000	8.2	4.372×10^5
1200	7.7	2.005×10^5
1400	7.3	1.047×10^5
1600	6.9	7.206×10^4

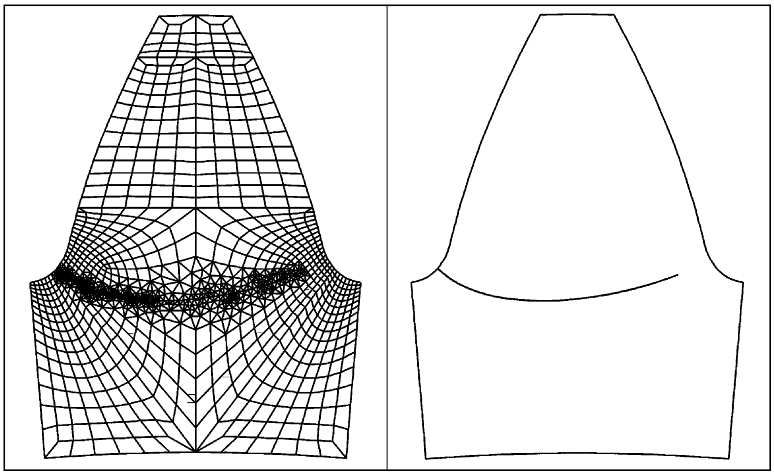

Fig. 9. Predicted crack propagation path in a gear tooth root.

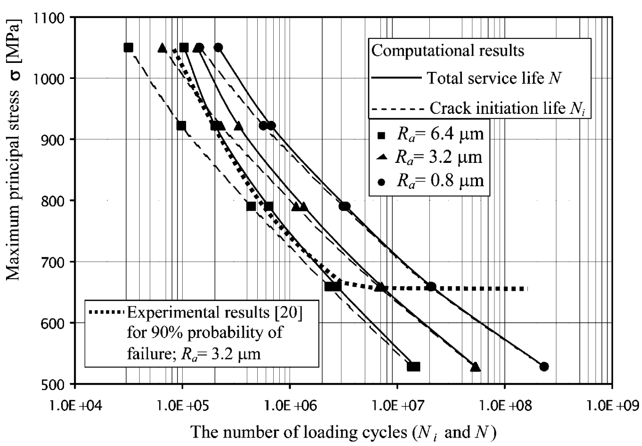

Fig. 10. The computed service life of treated spur gear and comparison with the available experimental results [20].

to tooth breakage in a tooth root is divided into crack initiation (N_i) and crack propagation (N_p) period, which enables the determination of total service life as $N = N_i + N_p$. The crack initiation period is based on stress–strain analysis in the framework of Finite Element Method, where it is assumed that the crack is initiated at the point of maximum principal stress in a gear tooth root. The displacement correlation method is then used for numerical determination of the functional relationship between the stress intensity factor and crack length $K = f(a)$, which is necessary for consequent analysis of fatigue crack growth.

The proposed model is used to determine the complete service life of spur gear made from high strength alloy steel 42CrMo4 (through-hardened). The computational analysis is carried out for variable loading and different surface finish of gear teeth flanks, which is characterised with the surface finish correction factor C_{sur} as a function of surface roughness and tensile strength of the material. The final results of the computational analysis are shown in Fig. 10, where two curves are presented for each surface finish of gear teeth flanks: the crack initiation curve and the curve of end of crack propagation, which at the same time represents the total service life. The results show that at low stress levels near fatigue limit almost all service life is spent in crack initiation. It is very important that cognition by determination of the service life of real gear drives in the engineering applications, because the majority of them really operate with loading conditions close to the fatigue limit. The computational results correspond well with the available experimental data [20].

The estimated bending fatigue life of the gear can deviate from real service life because some effects like non-homogenous material and possible causes of retardation of the crack propagation (crack closure) were not taken into account in the numerical analysis. Therefore, the model can be further improved with additional theoretical and numerical research, although additional experimental results will be required to provide the required material parameters.

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