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Travel time models for automated warehouses with aisle transferring storage and retrieval machine

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ABSTRACT

This paper presents analytical travel time models for the computation of travel time for automated warehouses with the aisle transferring S/R machine (in continuation multi-aisle AS/RS). These models consider the operating characteristics of the storage and retrieval machine such as acceleration and deceleration and the maximum velocity. Assuming uniform distributed storage rack locations and pick aisles and using the probability theory, the expressions of the cumulative distribution functions with which the mean travel time is calculated, have been determined. The computational models enable the calculation of the mean travel time for the single and dual command cycles, from which the performance of multiaisle AS/RS can be evaluated. A simulation model of multi-aisle AS/RS has been developed to compare the performances of the proposed analytical travel time models. The analyses show that regarding all examined types of multi-aisle AS/RS, the results of proposed analytical travel time models correlate with the results of simulation models of multi-aisle AS/RS.

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1. Introduction

Warehouses and distribution centers are an absolute necessity for a continuous and optimum operation of the production and distribution processes. A successful performance of a warehouse depends upon the appropriate design, layout and operation of the warehouse and material handling systems. When designing the warehouses a balance between flexibility, layout configuration, storage density and throughput capacity in order to achieve an effective design at a minimum cost have to be achieved. Estimates indicate that, depending on the type of industry, at least 25% of the cost of a product is represented by physical movement. Therefore every decision related to warehousing can reduce the logistics cost (Ramu, 1996; Tollazzi, 2006).

An important part of automated warehouses is represented by automated storage and retrieval systems (AS/RS). The basic components of AS/RS are storage racks (SR), storage and retrieval machines (S/R machines), input/output (I/O) locations and accumulating conveyors. Main advantages of the application of AS/RS are: efficient utilization of the warehouse space, reduction of damage and loss of goods, increased control upon storage and retrieval of goods and decrease in the number of warehouse workers. Due to the well known advantages of AS/RS (Tompkins et al., 2003) a high initial investment (approximately \$634,000 for a single-aisle AS/ RS, Zollinger, 1999) is necessary for the success of such systems. In the total initial investment in the automated warehouse alone S/R machines represent approximately 40% or more of the cost (Rosenblatt and Roll, 1993). The number of the S/R machines depends amongst other on the throughput capacity. When throughput capacity is high, the warehouse planners are obligated to prescribe the dedicated S/R machine in each pick aisle. On the other hand when throughput capacity is relatively low, multi-aisle AS/RS can be applied. In case of multi-aisle AS/RS, when the number of S/R machines is smaller than the number of pick aisles, a considerable saving of initial investment costs can be achieved. In order to evaluate the optimal number of S/R machines in multi-aisle AS/RS, the mean travel time for a storage and retrieval operation has to be determined.

Generally, AS/RS have been the subject of many researchers over the past few decades. Hausman et al. (1976), Graves et al. (1977) have presented travel time models for single-aisle AS/RS assuming that the SR is square-in-time (SIT). They have analyzed different storage strategies, e.g. randomized, turnover-based and class-based storage assignment rules. Gudehus (1973) has presented basic principles for determination of cycle times according to single-aisle AS/RS. With regard to other cycle time expressions, he has considered the impact of the acceleration and deceleration rate on travel times. Bozer and White (1984) have presented analytical model for the calculation of single command cycle (SC) and dual command cycle (DC) for non SIT racks. Their models are based on randomized storage and retrieval with different I/O

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configurations of the input queue. Hwang and Lee (1990), Vössner (1994), Vidovics (1994) and Wen et al. (2001) have presented travel time models considering the operating characteristics of the S/R machine for the single-aisle AS/RS and non SIT racks. Studies of multi-aisle AS/RS served by the single S/R machine have been presented by authors Hwang and Ko (1988). They have derived the travel time expression for multi-aisle AS/RS, assuming that the S/ R machine is transferred between adjacent aisles by a automatic transfer car (traverser) (the application of automatic aisle transferring S/R machine). They also investigate the problem of partitioning the aisles into a number of classes in order to minimize the number of S/R machines. Their study is based on travel time models considering average uniform velocity only. It must be emphasized that designing multi-aisle AS/RS just on average uniform velocity is far from being optimal from the practical point of view. Lerher et al. (2005, 2006) have presented analytical travel time models for multi-aisle AS/RS considering the operating characteristics of the storage and retrieval machine. In travel time expression the randomized storage assignment rule and the condition that the storage and retrieval processes can occur in the same picking aisle only, have been used. Since this is not in accordance with the practice, the above mentioned assumption has been released in the proposed travel time expressions. Sari et al. (2005) have presented the travel time models for the 3D flow-rack AS/RS. They have introduced the flow-rack, where transport unit load¹ (TUL) loaded by the S/R machine by one end of the rack, travels to another end of the rack to be retrieved. For the storage operation, the S/R machine operates in the same way as the S/R machine in the unit load AS/RS. However, the retrieval operation for a particular TUL requires that the S/R machine removes all TUL stored in front of the requested TUL. The acceleration/deceleration effect on the travel time was not considered by the authors. De Koster et al. (2008) have presented an optimal storage rack design for the 3D compact AS/RS. They have introduced the combination of the S/R machine for TUL movement in the horizontal and vertical directions and the system of inbound and outbound conveyors (powered or non-powered) for the depth movement. They have analyzed the system performance and optimal dimension of the system for the random storage strategy in order to minimize the mean travel time of the S/R machine. In the continuation of their work, Yu and De Koster (2009) have derived the mean single command cycle time under the full-turnover-based storage policy. In order to implement the class-based policy, Yu and De Koster (2009) have introduced the model for determining the optimal storage zone boundaries for the 3D compact AS/RS. For the determination of the zone boundaries, they have introduced the mixed-integer nonlinear model.

Gu et al. (2007) have presented a comprehensive review of research on warehouse operation. Roodbergen and Vis (2009) have presented a comprehensive explanation of the current state of the art in automated storage and retrieval systems. According to their papers the majority of the reviewed models used a different approach for analysing the system performance of warehouses from the one that will be used in our paper.

Therefore, our paper presents analytical travel time models for multi-aisle AS/RS considering (i) the operating characteristics of the S/R machine and (ii) the assumption that the storage and retrieval processes can occur in two randomly chosen pick aisles *i* and *j*. For the multi-aisle AS/RS the accomplishment of automatic aisle transferring S/R machine for pallets has been used. Because the layout of the multi-aisle AS/RS has a major influence on the performance and the efficiency of the warehouse, three different layouts of multi-aisle AS/RS with five picking aisles are presented. The proposed analytical travel time models have also been validated with discrete event simulations.

This paper is organized as follows: in the second section, the description of the multi-aisle AS/RS under study is given. In Section 3, the proposed analytical travel time models to calculate the mean travel time for the single and dual command cycle, considering the operating characteristics of the S/R machine are presented. The simulation model of multi-aisle AS/RS is presented in Section 4. In Section 5 the performance of the proposed models according to the simulation results is evaluated. Finally, the conclusion including a discussion of the application of proposed models is given in Section 6.

2. Multi-aisle AS/RS

A typical installation of multi-aisle AS/RS consists of a high-bay warehouse (pallet racking), with S/R machines operating in the cross warehouse aisle and in the pick aisle (Fig. 1). Installation heights of 22 m or more can be achieved, and typical operating (pick) aisles for standard euro pallets can be about 1.5 m wide. The warehouse management system monitors the status of all components in the system and, based on the warehouse inventory and movement requirements, it plans the work to be carried out. The S/R machine that works within the multi-aisle AS/RS consists of a vertical mast or a pair of masts supporting the hoisted carriage, which can be raised or lowered. The S/R machine travels on the floor-mounted rail with an overhead guide rail. The amount of required storage racking depends on the designed inventory holding capacity. The required number of S/R machines is determined by the total amount of TUL movement in a given period of time (throughput capacity of the system). If the number of S/R machines is significantly lower than the number of pick aisles, a transfer facility with one or more transfer cars on to which the S/R machines can be driven and moved between adjacent pick aisles could be incorporated into the design of automated warehouse. An alternative method is to curve the rails at the end of pick aisles, so that the S/R machine can run down the cross warehouse aisle (Rushton et al., 2006). Many producers of the warehouse equipment, such as Siemens Dematic and Stöcklin have begun to offer such systems served by automatic curve going or automatic aisle transferring S/R machines. The main benefits using multi-aisle AS/RS are: subsequent expansion of S/R machines is possible at any time; optimum use is made of space due to minimal overrun dimensions and high throughput resulting from pallet buffer positions on the transfer car (application of the automatic aisle transferring S/R machines).

When developing the multi-aisle AS/RS, the following assumptions were considered (Lerher, 2005, 2006):

- The multi-aisle AS/RS is considered to be divided into pick aisles with SR on both sides, thus there are double SR between pick aisles *i* and single SR along the warehouse walls. The I/O_{warehouse} location of multi-aisle AS/RS is located in the beginning of the cross warehouse aisle and is perpendicular to the pick aisle 1 (Fig. 1).
- The sequences of (i) acceleration, constant velocity and deceleration and (ii) acceleration and deceleration have been used.
- The S/R machine is able to travel in cross warehouse aisle through a transfer car called "traverser", so that it can enter any pick aisle *i*.
- The SR is considered to be a continuous rectangular pick face where the I/O_{aisle(i)} location is located in the cross warehouse aisle and is perpendicular to the pick aisle *i*.
- The S/R machine operates either on single command cycle or dual command cycle.

¹ TUL stands for the assembly of individual packages or items on a stock keeping unit – pallet for the efficient handling by mechanical equipment.



Fig. 1. Multi-aisle AS/RS with aisle transferring S/R machine.

- The specification of the S/R machine (max. velocities in horizontal and vertical directions and acceleration and deceleration rates) as well as the length and the height of the SR are known.
- The S/R machine travels simultaneously in the horizontal and vertical directions in the pick aisle.
- The length and the height of the SR are long enough for the S/R machine to reach max. velocity from the I/O_{aisle(i)} location.
- The width of the cross-aisle is wide (long) enough for the "traverser" to reach max. velocity from the I/O_{war} location.
- The randomized storage assignment policy is used. That is, any point (storage location) within the SR is equally likely to be selected for the storage or retrieval request.

Also the following notation is introduced:

AS/RS	automated storage and retrieval system
SR	storage rack
COM	storage compartment
I/O	input/output location
$I/O_{aisle(i)}$	input/output location of the pick aisle <i>i</i>
I/O _{war}	input/output location of the warehouse
TUL	transport unit load
SC	single command cycle
DC	dual command cycle
FCFS	first-come-first-serve selection rule
MASS	multi-aisle AS/RS
Р	probability
F(t)	cumulative distribution function
f(t)	probability density function
E(TBA)	expected travel-between aisles time component
$E_k(SC)_{MASS}$	expected single command travel time for the multi-
	aisle AS/RS corresponding to the <i>k</i> th condition

- *E*(TB) expected travel-between time for dual command cycle
- $E_k(TB_1)$ expected travel-between time for dual command cycle corresponding to the *k*th condition and the condition *i* = *j*
- $E_k(TB_2)$ expected travel-between time for dual command cycle corresponding to the *k*th condition and the condition $i \neq j$
- $E_k(DC)_{MASS}$ expected dual command travel time for the multiaisle AS/RS corresponding to the *k*th condition
- $T(DC)_{MASS}$ mean dual command travel time for the multi-aisle AS/RS corresponding to the *k*th condition v(t) velocity of the S/R machine at time *t*
- $v(t_p)$ peak velocity of the S/R machine at time t_p
- v_{max} maximum velocity of the S/R machine
- v_x maximum velocity of the S/R machine in the
- horizontal (x) direction v_v maximum velocity of the hoisted carriage in the
 - vertical (y) direction
- v_i maximum velocity of the "traverser" in the cross (z) direction
- *a* acceleration/deceleration of the S/R machine *a*_x acceleration/deceleration of the S/R machine in the
- *a_x* acceleration/deceleration of the S/R machine in the horizontal (*x*) direction
- *a_y* acceleration/deceleration of the hoisted carriage in the vertical (*y*) direction
- a_i acceleration/deceleration of the "traverser" in the cross (z) direction
- d(T) distance moved during time t
- t_p time necessary to reach the peak velocity
- *T* arrival time at a destination
- *L* length of the SR

S_X	additional distance of the length in the cross
	warehouse aisle
Н	height of the SR
W	width of the cross warehouse aisle
l	minimum distance necessary to reach v_x
h	minimum distance necessary to reach v_{y}
w	minimum distance necessary to reach v_i
t_x	required travel time for the S/R machine to reach <i>l</i>
t _{sx}	travel time for the S/R machine for travelling of the
	additional distance s_x
T_x	required travel time for the S/R machine to reach L
t_{v}	required travel time for the hoisted carriage to reach h
\tilde{T}_{v}	required travel time for the hoisted carriage to reach
5	H
t _i	required travel time for the traverser in the cross
	warehouse aisle to reach <i>w</i>
T _i	required travel time for the traverser in the cross
	warehouse aisle to reach W
<i>a</i> (<i>i</i>)	required travel time for transferring of the S/R
	machine from the I/O _{war} to the I/O _{aisle(i)}
a(j)	required travel time for transferring of the S/R
	machine from the I/O _{war} to the I/O _{aisle(j)}
A(i, j)	required travel time for transferring of the S/R
	machine from the I/O _{aisle(i)} to the I/O _{aisle(j)}
W _{aisle}	width of the pick aisle
$W_{M=i}$	width of the cross warehouse aisle according to the
	ith number of picking aisles
l _{COM}	length of the storage compartment
h _{COM}	height of the storage compartment
N _x	number of storage compartments in the horizontal
	direction
Ny	number of storage compartments in the vertical
	direction
n	number of TUL in the storage compartment
w, g	width of the pallet, length of the pallet
b_1, b_2	safety addition to the width of the storage
	compartment, safety addition to the height of the
	storage compartment

b_{3}, b_{4}	width of the upright frame, height of the rack beam
b_{5}, b_{6}	deviation of the storage compartment from the floor,
	safety spacing between storage racks that are placed
	close to each other
Μ	number of pick aisles
Y	number of storage racks
Q	warehouse volume (rack capacity)
1	performance comparison between continuous models

performance comparison between continuous models λ and the discrete model

3. Analytical travel time models for multi-aisle AS/RS

3.1. The fundamentals of travel time

Two types of velocity profiles can be distinguished depending on whether the obtained peak velocity $v(t_p)$ is less than v_{max} (type I) or equal to v_{max} (type II) (Fig. 2). It can be verified that time $T < 2v_{max}/a$ for type I and $T > 2v_{max}/a$ for type II.

3.1.1. S/R machine travelling for type I ($T < 2v_{max}/a$)

The velocity in dependence of time v(t) equals the following expression:

$$v(t) = \begin{cases} at, & t \in (0, t_p) \\ -a(t-T), & t \in (t_p, T) \end{cases}$$
(1)

The distance in dependence of time d(T) equals the following expression:

$$d(T) = \int_0^T v(t)dt = \frac{a \cdot T^2}{4} \tag{2}$$

Because of the acceleration and deceleration are equal in magnitude, the time necessary to reach the peak velocity equals $t_p = T/2$. For the verification of the expression 2 see Appendix A.

3.1.2. S/R machine travelling for type II $(T > 2v_{max}/a)$

The velocity in dependence of time v(t) equals the following expression:



Fig. 2. Velocity-time relationship of the S/R machine.

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____ Dual command cycle in multi-aisle AS/RS

Fig. 3. The single and dual command cycle in multi-aisle AS/RS.

$$v(t) = \begin{cases} at, & t \in (0, t_p) \\ v_{max}, & t \in (t_p, T - t_p) \\ -a(t - T), & t \in (T - t_p, T) \end{cases}$$
(3)

The distance in dependence of time d(T) equals the following expression:

$$d(T) = \int_0^T v(t)dt = v_{max} \cdot T - \frac{v_{max}^2}{a}$$
(4)

For the verification of the expression 4 see Appendix A.

3.2. Single command cycle in multi-aisle AS/RS

The operation of the SC encompasses either the storage or the retrieval sequence. With regard to the single-aisle AS/RS, the SC in the multi-aisle AS/RS combines transferring in the cross warehouse aisle and travelling of the S/R machine in the *i*th pick aisle (Fig. 3). The efficiency of the SC in the multi-aisle AS/RS is therefore based on:

- transferring of the S/R machine to adjacent pick aisle through cross warehouse aisle,
- travelling of the S/R machine in *i*th pick aisle.

3.2.1. Transferring of the S/R machine to adjacent pick aisle

Travelling between aisles time component (TBA) corresponds to transferring of the S/R machine through traverser from I/O_{war} location to the *i*th I/O_{aisle} location (see the dashed line in Fig. 3). According to the assumption that only the S/R machine is transferring through a traverser in the cross warehouse aisle while the hoisted carriage stays still, the horizontal movement of the traverser in the cross warehouse aisle has been considered. As regards the condition of uniform distribution of I/O_{aisle} locations in the cross warehouse aisle, the cumulative distribution function $F_i(t)$

is accomplished. The cumulative distribution function $F_i(t)$ is distinguished according to the following condition:

- (i) transferring of the S/R machine for type I, where $t_i = 2v_i/a_i$ is the required travel time for the traverser in the cross warehouse aisle to reach $w = v_i^2/a_i$.
- (ii) transferring of the S/R machine for type II, where $T_i = W/v_i + v_i/a_i$ is the required travel time for the traverser in the cross warehouse aisle to reach *W*.
- Cumulative distribution function $F_i(t)$ for transferring of the S/R machine in the cross warehouse aisle

$$F_{i}(t) = \begin{cases} \frac{t^{2}a_{i}}{4W}, & \left(0 \leqslant t \leqslant \frac{2v_{i}}{a_{i}}\right) \\ \frac{v_{i}t}{W} - \frac{v_{i}^{2}}{a_{i}W}, & \left(\frac{2v_{i}}{a_{i}} \leqslant t \leqslant \frac{W}{v_{i}} + \frac{v_{i}}{a_{i}}\right) \end{cases}$$
(5)

• Cumulative distribution function *F*(*t*)

The cumulative distribution function F(t) depends on the relationships among the values of the following parameters: v_i, a_i, W . Therefore F(t) can be specified under the following condition:

$$F(t) = F_i(t), \quad (0 \le t \le T_i) \tag{6}$$

The expected one way travel time E(ES) for transferring of the S/ R machine in the cross warehouse aisle is equal to the next expression:

$$E(\text{ES}) = \int_0^{t_i} (1 - F(t)) dt$$
(7)

Now, the expected travel-between aisles time component E(TBA) becomes:

$$E(TBA) = 2 \cdot E(ES) \tag{8}$$

3.2.2. Travelling of the S/R machine in pick aisle i

Under travelling of the S/R machine in the *i*th pick aisle, the S/R machine is capable of visiting a single storage or retrieval location. The travel time depends on the kinematics properties of the S/R machine and the hoisted carriage, the length and the height of the SR and the selected storage assignment rule. According to the work of Hwang and Lee (1990), Vössner (1994) and Vidovics (1994), the variable travel time t_{xy} from the I/O_{aisle(i)} location to any randomly selected location in the *i*th pick aisle is the maximal value of t_x or t_y , where t_x is the horizontal travel time and t_y is the vertical travel time. As regards the condition of uniform distribution of storage locations in the SR and the condition of the *x*-coordinate and *y*-coordinate independence, the cumulative distribution functions $F_x(t)$ and $F_y(t)$ are accomplished. The cumulative distribution functions $F_x(t)$ and $F_y(t)$ are distinguished according to the following condition:

- (i) S/R machine travelling and movement of the hoisted carriage for type I, where $t_x = 2v_x/a_x$ and $t_y = 2v_y/a_y$ are required travel times for the S/R machine and the hoisted carriage to reach $l = v_x^2/a_x$ and $h = v_y^2/a_y$. Travel time of the S/R machine for travelling of the additional distance s_x in the cross warehouse aisle, is expressed with $t_{sx} = s_x/v_x$.
- (ii) S/R machine travelling and movement of the hoisted carriage for type II, where $T_x = L/v_x + v_x/a_x$ and $T_y = H/v_y + v_y/a_y$ are required travel times for the S/R machine and the hoisted carriage to reach *L* and *H*.

• Cumulative distribution function $F_x(t)$ for travelling of the S/ R machine in the horizontal direction

$$F_{x}(t) = \begin{cases} \frac{a_{x}t^{2}}{4L}, & \left(0 \leqslant t \leqslant \frac{s_{x}}{v_{x}}\right) \\ \frac{a_{x}t^{2}}{4L}, & \left(0 \leqslant t \leqslant \frac{2v_{x}}{a_{x}}\right) \\ \frac{v_{x}t}{L} - \frac{v_{x}^{2}}{a_{x}L}, & \left(\frac{2v_{x}}{a_{x}} \leqslant t \leqslant \frac{t}{v_{x}} + \frac{v_{x}}{a_{x}}\right) \end{cases}$$
(9)

• Cumulative distribution function $F_y(t)$ for movement of the hoisted carriage in the vertical direction

$$F_{y}(t) = \begin{cases} \frac{a_{y}t^{2}}{4H}, & \left(0 \leqslant t \leqslant \frac{2v_{y}}{a_{y}}\right) \\ \frac{v_{y}t}{H} - \frac{v_{y}^{2}}{a_{y}H}, & \left(\frac{2v_{y}}{a_{y}} \leqslant t \leqslant \frac{H}{v_{y}} + \frac{v_{y}}{a_{y}}\right) \end{cases}$$
(10)

• Cumulative distribution function *F*(*t*)

The cumulative distribution function F(t) is defined according to the travelling of the S/R machine in the horizontal direction and the movement of the hoisted carriage in the vertical direction and depends on the relationships among the values of the following parameters: v_x , v_y , a_x , a_y , L, H. Therefore F(t) can be specified with theoretically six different cases (k = 1, ..., 6).

$$F_k(t) = F_x(t) \cdot F_y(t), \quad (0 \le t \le T)$$
(11)

The expected one way travel time under single command cycle $E_k(EP)$, corresponding to the *k*th condition is equal to the next expression:

$$E_k(\text{EP}) = \int_0^{\max.(T_x,T_y)} (1 - F_k(t)) dt, \quad k = (1, \dots, 6)$$
(12)

Now, the expected single command travel time $E_k(SC)$ in picking aisle *i* corresponding to the *k*th condition becomes:

$$E_k(SC) = 2 \cdot E_k(EP) \tag{13}$$

For a detailed representation of the cumulative distribution functions, which deals with the above mentioned conditions, see papers from Hwang and Lee (1990), Vössner (1994) and Vidovics (1994).

3.2.3. Expected single command travel time for multi-aisle AS/RS

According to transferring of the S/R machine to the adjacent pick aisle and travelling of the S/R machine in the pick aisle *i*, the expected single command travel time for the multi-aisle AS/RS corresponding to the *k*th condition is represented with the following expression:

$$E_k(SC)_{MASS} = E_k(SC) + E(TBA)$$
(14)

3.3. Dual command cycle in multi-aisle AS/RS when i = j

The dual command cycle makes the S/R machine visit up to two locations between successive returns to the I/O_{war} location. After completing a given storage request, the S/R machine can move directly to another location for the next retrieval request without returning to the I/O_{war} location. Therefore the travel time for DC corresponds to the travel time for SC in the randomly selected pick aisle and travel-between time for DC, where the retrieval request occurs in the same pick aisle *i* or *j*, with the condition i = j (Fig. 3).

By definition DC involves two randomly selected locations in the pick aisle *i*; one representing the storage point $P_1(x_1, y_1)$ and the other representing the retrieval point $P_2(x_2, y_2)$. The expected travel-between (TB) time for DC for two randomly selected points in the pick aisle *i* is the same as $E_k(TB_1)$, in which *k* is determined by the given SR configuration and the S/R machine technical configuration. According to the condition of uniform distribution of storage locations in the SR and the condition of the *x*-coordinate and *y*-coordinate independence, the cumulative distribution functions $F_x(t)$ and $F_y(t)$ are accomplished. The cumulative distribution functions are distinguished according to the following condition:

- (i) S/R machine travelling and movement of the hoisted carriage for type I, where $t_x = 2v_x/a_x$ and $t_y = 2v_y/a_y$ are required travel times for the S/R machine and the hoisted carriage to reach $l = v_x^2/a_x$ and $h = v_y^2/a_y$.
- (ii) S/R machine travelling and movement of the hoisted carriage for type II, where $T_x = L/v_x + v_x/a_x$ and $T_y = H/v_y + v_y/a_y$ are required travel times for the S/R machine and the hoisted carriage to reach *L* and *H*.
- Cumulative distribution function $F_x(t)$ for travelling of the S/ R machine in the horizontal direction

$$F_{x}(t) = \begin{cases} \frac{a_{x}}{2L}t^{2} - \frac{a_{x}^{2}}{16L^{2}}t^{4}, & \left(0 \leqslant t \leqslant \frac{2v_{x}}{a_{x}}\right) \\ -\frac{v_{x}^{2}}{L^{2}}t^{2} + \left[\frac{2v_{x}^{2}}{a_{x}L^{2}} + \frac{2v_{x}}{L}\right]t - \frac{2v_{x}^{2}}{a_{x}L} - \frac{v_{x}^{4}}{a_{x}^{2}L^{2}}, & \left(\frac{2v_{x}}{a_{x}} \leqslant t \leqslant \frac{L}{v_{x}} + \frac{v_{x}}{a_{x}}\right) \end{cases}$$
(15)

- Cumulative distribution function $F_y(t)$ for movement of the hoisted carriage in the vertical direction

$$F_{y}(t) = \begin{cases} \frac{a_{y}}{2H}t^{2} - \frac{a_{y}^{2}}{16H^{2}}t^{4}, & \left(0 \leqslant t \leqslant \frac{2v_{y}}{a_{y}}\right) \\ -\frac{v_{y}^{2}}{H^{2}}t^{2} + \left[\frac{2v_{y}^{3}}{a_{y}H^{2}} + \frac{2v_{y}}{H}\right]t - \frac{2v_{y}^{2}}{a_{y}H} - \frac{v_{y}^{4}}{a_{y}^{2}H^{2}}, & \left(\frac{2v_{y}}{a_{y}} \leqslant t \leqslant \frac{H}{v_{y}} + \frac{v_{y}}{a_{y}}\right) \end{cases}$$
(16)

• Cumulative distribution function F(t)

The cumulative distribution function F(t) is defined according to the travelling of the S/R machine in the horizontal direction and the movement of the hoisted carriage in the vertical direction and depends on the relationships among the values of the following parameters: v_x , v_y , a_x , a_y , L, H. Therefore F(t) can be specified with theoretically six different cases (k = 1, ..., 6).

$$F_k(t) = F_x(t) \cdot F_y(t), \quad (0 \le t \le T)$$
(17)

The expected travel-between time $E_k(\text{TB}_1)$ for DC for two randomly selected points corresponding to the *k*th condition (k = 1, ..., 6) is equal to the following expression:

$$E_k(\mathrm{TB}_1) = \int_0^{\max \cdot (T_x, T_y)} (1 - F_k(t)) dt, \quad k = (1, \dots, 6)$$
(18)

For a detailed representation of the cumulative distribution functions, which deals with the above mentioned conditions, see papers from Hwang and Lee (1990), Vössner (1994) and Vidovics (1994).

3.3.1. Expected dual command travel time for multi-aisle AS/RS when i = j

According to travelling of the S/R machine in the horizontal direction and movement of the hoisted carriage in the vertical direction, the expected dual command travel time for the multiaisle AS/RS corresponding to the *k*th condition and the condition i = j is represented with the following expression:

$$E_k(DC)_{MASS} = E_k(SC)_{MASS} + E_k(TB_1)$$
(19)

3.4. Dual command cycle in multi-aisle AS/RS when $i \neq j$

In practice it could happen that the storage request is performed in the pick aisle *i*, while the retrieval request is performed in pick aisle *j*. In that case $i \neq j$ must be considered when deriving the travel-between time (TB₂) for DC. When developing the analytical models, the procedure of geometrics probability has been considered (Gnedenko, 1969). It must be emphasized that beside the assumption of uniform velocity for travelling of the S/R machine, the acceleration and deceleration have also been considered (Lerher, 2005).

Let the distribution function F(t) represent the probability that the TB₂ for DC for two randomly chosen locations in the pick aisle *i* and *j* is less than or equal to *t*. This statement can be described as follows:

$$F(t) = P(|t_{x_1} - t_{x_2}| \le t) \cdot P(|t_{y_1} - t_{y_2}| \le t)$$
(20)

In continuation the movement of the hoisted carriage in the vertical direction and travelling and transferring of the S/R machine in the horizontal direction (pick aisle and cross warehouse aisle) will be considered separately.

3.4.1. Movement of the hoisted carriage in the vertical direction

The cumulative distribution function $F_y(t)$, which represents movement of the hoisted carriage in the vertical direction, is equal to the following expression.

• Cumulative distribution function $F_y(t)$ for movement of the hoisted carriage in the vertical direction

$$P(|t_{y1} - t_{y2}| \leq t) = \begin{cases} \underbrace{\frac{a_y}{2H}t^2 - \frac{a_y^2}{16H^2}t^4}_{F_{y1}}, \\ 0 \leq t \leq \frac{2v_y}{a_y} \\ -\frac{v_y^2}{H^2}t^2 + \left[\frac{2v_y^3}{a_yH^2} + \frac{2v_y}{H}\right]t - \frac{2v_y^2}{a_yH} - \frac{v_y^4}{a_y^2H^2}, \\ \underbrace{\frac{2v_y}{a_y} \leq t \leq \frac{H}{v_y} + \frac{v_y}{a_y}}_{F_{y2}} \end{cases}$$

$$(21)$$

where $t_y = 2v_y/a_y$ is the required travel time for the hoisted carriage to reach $h = v_y^2/a_y$ and $T_y = H/v_y + v_y/a_y$ is the required travel time for the hoisted carriage to reach *H*.

For a detailed representation of the cumulative distribution function $F_y(t)$, see papers from Hwang and Lee (1990), Vössner (1994) and Vidovics (1994).

3.4.2. Travelling and transferring of the S/R machine in the horizontal direction

The distribution function $F_x(t)$, which represents travel time for travelling and transferring of the S/R machine in the horizontal direction, is equal to the following expression:

$$P(T_{x1} + T_{x2} + A(i,j) \le t)$$
(22)

where:

- $T_{x1} = L/v_x + v_x/a_x$ is the required travel time for the S/R machine to reach *L* in the pick aisle *i*.
- $T_{x2} = L/v_x + v_x/a_x$ is the required travel time for the S/R machine to reach *L* in the pick aisle *j*.



Fig. 4. Travelling of the S/R machine in the horizontal direction.

- A(i,j) = |a(i) a(j)| is the required travel time for transferring of the S/R machine from the I/O_{aisle(i)} to the I/O_{aisle(i)}.
- a(i) and a(j) are required travel times for transferring of the S/R machine from the I/O_{war} to the I/O_{aisle(i)} and I/O_{aisle(j)}.

Travelling of the S/R machine based on two following assumptions (Fig. 4):

• Condition 1: $0 \leq t \leq T_x$

The total travel time for travelling of the S/R machine in the horizontal direction between two randomly chosen locations in the pick aisles *i* and *j* is less than or equal to the maximal travel time for travelling of the S/R machine in any of the pick aisle (Fig. 4 – condition 1).

$$P(T_{x1} + T_{x2} \leq t) = \frac{1}{\left(\frac{L}{\nu_x} + \frac{\nu_x}{a_x}\right)^2} \left[\int_0^t dx_1 \cdot \int_0^{-x_1 + t} dx_2 \right]$$
$$= \frac{t^2}{2\left(\frac{L}{\nu_x} + \frac{\nu_x}{a_x}\right)^2}$$
(23)

• Condition 2: $T_x \leq t \leq 2T_x$

The total travel time for travelling of the S/R machine in the horizontal direction between two randomly chosen locations in the pick aisles *i* and *j* is larger than or equal to the maximal travel time for travelling of the S/R machine in any of the pick aisle; and at the same time is less than or equal to the double maximal travel time for travelling of the S/R machine in any of the pick aisle (Fig. 4 – condition 2).

$$P(T_{x1} + T_{x2} \leqslant t) = \frac{1}{\left(\frac{v_x}{a_x} + \frac{1}{v_x}\right)^2} \left[\int_0^{t - \left(\frac{v_x}{a_x} + \frac{1}{v_x}\right)} dx_1 \cdot \int_0^{\left(\frac{v_x}{a_x} + \frac{1}{v_x}\right)} dx_2 + \int_{t - \left(\frac{v_x}{a_x} + \frac{1}{v_x}\right)}^{\left(\frac{v_x}{a_x} + \frac{1}{v_x}\right)} dx_1 \cdot \int_0^{-x_1 + t} dx_2 \right]$$
(24)

$$P(T_{x1} + T_{x2} \leq t) = \frac{1}{2} \left(-2 + \frac{ta_x v_x (4v_x^2 + a_x (4L - tv_x))}{(La_x + v_x^2)^2} \right)$$

Besides the time for travelling of the S/R machine in the pick aisle we have to consider the time for transferring of the S/R machine to the adjacent aisle in the cross warehouse aisle A(i,j) = |a(i) - a(j)|, which is described by the following expression (Fig. 5):

$$P(T_{x1} + T_{x2} + A(i,j) \le t) = P(T_{x1} + T_{x2} \le t - A(i,j))$$
(25)

Considering travelling in the pick and transferring in the cross warehouse aisle, the above stated conditions 1 and 2 are expressed as follows:

• Condition 1:
$$0 \leq t - A(i,j) \leq T_x \Rightarrow A(i,j) \leq t \leq A(i,j) + T_x$$

$$P(A(i,j) \le t \le A(i,j) + T_x) = \frac{A(i,j)^2 - 2A(i,j)t + t^2}{2\left(\frac{v_x}{a_x} + \frac{L}{v_x}\right)^2}$$
(26)

• Condition 2:
$$T_x \leq t - A(i,j) \leq 2T_x \Rightarrow A(i,j) + T_x \leq t \leq A(i,j) + 2T_x$$

$$P(A(i,j) + T_x \leq t \leq A(i,j) + 2T_x) = \frac{1}{2} \left(-2 + \frac{(t - A(i,j))a_xv_x(4v_x^2 + a_x(4L - (t - A(i,j))v_x))}{(La_x + v_x^2)^2} \right)$$
(27)

The cumulative distribution function $F_x(t)$, which represents travelling in the horizontal direction in the pick aisles *i* and *j* and transferring in the cross warehouse aisle, is represented by the following expression.

• Cumulative distribution function $F_x(t)$ for travelling and transferring of the S/R machine in the horizontal direction

$$F_{x}(t) = \begin{cases} \underbrace{\frac{A(i,j)^{2} - 2A(i,j)t + t^{2}}{2\left(\frac{v_{x}}{a_{x}} + \frac{L}{v_{x}}\right)^{2}},} \\ \underbrace{\frac{2\left(\frac{v_{x}}{a_{x}} + \frac{L}{v_{x}}\right)^{2}}{F_{x1}},} \\ A(i,j) \leqslant t \leqslant A(i,j) + T_{x} \\ \underbrace{\frac{1}{2}\left(-2 + \frac{(t - A(i,j))a_{x}v_{x}(4v_{x}^{2} + a_{x}(4L - (t - A(i,j))v_{x})))}{(La_{x} + v_{x}^{2})^{2}}\right),} \\ \underbrace{\frac{1}{2}\left(-2 + \frac{(t - A(i,j))a_{x}v_{x}(4v_{x}^{2} + a_{x}(4L - (t - A(i,j))v_{x})))}{(La_{x} + v_{x}^{2})^{2}}\right),} \\ \underbrace{\frac{1}{2}\left(-2 + \frac{(t - A(i,j))a_{x}v_{x}(4v_{x}^{2} + a_{x}(4L - (t - A(i,j))v_{x}))}{(La_{x} + v_{x}^{2})^{2}}\right),} \\ \underbrace{\frac{1}{2}\left(-2 + \frac{(t - A(i,j))a_{x}v_{x}(4v_{x}^{2} + a_{x}(4L - (t - A(i,j))v_{x}))}{(La_{x} + v_{x}^{2})^{2}}\right),} \\ \underbrace{\frac{1}{2}\left(-2 + \frac{(t - A(i,j))a_{x}v_{x}(4v_{x}^{2} + a_{x}(4L - (t - A(i,j))v_{x}))}{(La_{x} + v_{x}^{2})^{2}}\right),} \\ \underbrace{\frac{1}{2}\left(-2 + \frac{(t - A(i,j))a_{x}v_{x}(4v_{x}^{2} + a_{x}(4L - (t - A(i,j))v_{x}))}{(La_{x} + v_{x}^{2})^{2}}\right),} \\ \underbrace{\frac{1}{2}\left(-2 + \frac{(t - A(i,j))a_{x}v_{x}(4v_{x}^{2} + a_{x}(4L - (t - A(i,j))v_{x})}{(La_{x} + v_{x}^{2})^{2}}\right),} \\ \underbrace{\frac{1}{2}\left(-2 + \frac{(t - A(i,j))a_{x}v_{x}(4v_{x}^{2} + a_{x}(4L - (t - A(i,j))v_{x})}{(La_{x} + v_{x}^{2})^{2}}\right),} \\ \underbrace{\frac{1}{2}\left(-2 + \frac{(t - A(i,j))a_{x}v_{x}(4v_{x}^{2} + a_{x}(4L - (t - A(i,j))v_{x})}{(La_{x} + v_{x}^{2})^{2}}\right),} \\ \underbrace{\frac{1}{2}\left(-2 + \frac{(t - A(i,j))a_{x}v_{x}(4v_{x}^{2} + a_{x}(4L - (t - A(i,j))v_{x})}{(La_{x} + v_{x}^{2})^{2}}\right),} \\ \underbrace{\frac{1}{2}\left(-2 + \frac{(t - A(i,j))a_{x}v_{x}(4v_{x}^{2} + a_{x}(4L - (t - A(i,j))v_{x})}{(La_{x} + v_{x}^{2})^{2}}\right),} \\ \underbrace{\frac{1}{2}\left(-2 + \frac{(t - A(i,j))a_{x}v_{x}(4v_{x}^{2} + a_{x}(4L - (t - A(i,j))v_{x})}{(La_{x} + v_{x}^{2})^{2}}\right),} \\ \underbrace{\frac{1}{2}\left(-2 + \frac{(t - A(i,j))a_{x}v_{x}(4v_{x}^{2} + a_{x}(4L - (t - A(i,j))v_{x}}{(La_{x} + v_{x}^{2})^{2}}\right),} \\ \underbrace{\frac{1}{2}\left(-2 + \frac{(t - A(i,j))a_{x}v_{x}(4v_{x}^{2} + a_{x}(4L - (t - A(i,j))v_{x}}{(La_{x} + v_{x}^{2})^{2}}\right),} \\ \underbrace{\frac{1}{2}\left(-2 + \frac{(t - A(i,j))a_{x}v_{x}(4v_{x}^{2} + a_{x}(4L - (t - A(i,j))v_{x}}{(La_{x} + v_{x}^{2})^{2}}\right),} \\ \underbrace{\frac{1}{2}\left(-2 + \frac{(t - A(i,j))a_{x}v_{x}(4v_{x} + a_{x}(4v_{x} + v_{x}^{2})v_{x}}{(La_{x} + v_{x}^{2})}\right),$$

3.4.3. Cumulative distribution function F(t)

The cumulative distribution function F(t) is defined according to the travelling and transferring of the S/R machine in the horizontal direction $F_x(t)$ and the movement of the hoisted carriage in the vertical direction $F_y(t)$ and it holds for the following conditions (Lerher, 2005):

• Condition 1: $T_y \leq A(i,j)$

The required travel time for the movement of the hoisted carriage from the storage request to the retrieval request is less than or equal to the required travel time for transfer the S/R machine from the *i*th pick aisle to the *j*th pick aisle.

The probability density function $f_1(t)$ becomes:

$$f_{1}(t) = \begin{cases} F'_{x1}, & A(i,j) \leq t \leq A(i,j) + T_{x} \\ F'_{x2}, & A(i,j) + T_{x} \leq t \leq A(i,j) + 2T_{x} \end{cases}$$
(29)

The expected travel-between time $E_1(TB_2)$ for DC corresponding to the 1st condition becomes:

$$E_1(\text{TB}_2) = \int_{A(i,j)}^{A(i,j)+2T_x} t \cdot f_1(t) dt$$
(30)

• **Condition 2:** $A(i,j) \leq T_y \leq A(i,j) + T_x$

The required travel time for the movement of the hoisted carriage from the storage request to the retrieval request is larger than or equal to the required travel time for the transfer of the S/R machine from the *i*th pick aisle to the *j*th pick aisle. At the same time it is smaller than or equal to the sum of required travel times for transfer of the S/R machine from the *i*th pick aisle to the *j*th pick aisle and travelling of the S/R machine in any of the pick aisle.

The probability density function $f_2(t)$ becomes:



Fig. 5. Transferring of the S/R machine between the *i*th and *j*th aisles.

$$f_{2}(t) = \begin{cases} F'_{x1} \cdot F_{y1} + F_{x1} \cdot F'_{y1}, & A(i,j) \leq t \leq T_{y} \\ F'_{x1}, & T_{y} \leq t \leq A(i,j) + T_{x} \\ F'_{x2}, & A(i,j) + T_{x} \leq t \leq A(i,j) + 2T_{x} \end{cases}$$
(31)

The expected travel-between time $E_2(TB_2)$ for DC corresponding to the 2nd condition becomes:

$$E_{2}(TB_{2}) = \int_{A(i,j)}^{A(i,j)+2T_{x}} t \cdot f_{2}(t) dt$$
(32)

• **Condition 3:** $A(i,j) + T_x \leq T_y \leq A(i,j) + 2T_x$

The required travel time for the movement of the hoisted carriage from the storage request to the retrieval request is larger than or equal to the required travel time for the transfer of the S/R machine from the *i*th pick aisle to the *j*th pick aisle and travelling of the S/R machine in any of the pick aisle. At the same time is smaller than or equal to the sum of required travel times for the transfer of the S/R machine from the *i*th pick aisle to the *j*th pick aisle and double travelling of the S/R machine in any of the pick aisle.

The probability density function $f_3(t)$ becomes:

$$f_{3}(t) = \begin{cases} F'_{x1} \cdot F_{y1} + F_{x1} \cdot F'_{y1}, & A(i,j) \leq t \leq A(i,j) + T_{x} \\ F'_{x2} \cdot F_{y1} + F_{x2} \cdot F'_{y1}, & A(i,j) + T_{x} \leq t \leq T_{y} \\ F'_{x2}, & T_{y} \leq t \leq A(i,j) + 2T_{x} \end{cases}$$
(33)

The expected travel-between time $E_3(TB_2)$ for DC corresponding to the 3rd condition becomes:

$$E_{3}(TB_{2}) = \int_{A(ij)}^{A(ij)+2T_{x}} t \cdot f_{3}(z) dz$$
(34)

• Condition 4: $T_y \ge A(i,j) + 2T_x$

The required travel time for the movement of the hoisted carriage from the storage request to the retrieval request is larger than or equal to the required travel time for the transfer of the S/R machine from the *i*th pick aisle to the *j*th pick aisle and double travelling of the S/R machine in any of the pick aisle.

The probability density function $f_4(t)$ becomes:

$$f_{4}(t) = \begin{cases} F'_{x1} \cdot F_{y1} + F_{x1} \cdot F'_{y1}, & A(i,j) \leqslant t \leqslant A(i,j) + T_{x} \\ F'_{x2} \cdot F_{y1} + F_{x2} \cdot F'_{y1}, & A(i,j) + T_{x} \leqslant t \leqslant A(i,j) + 2T_{x} \\ F'_{y2}, & A(i,j) + 2T_{x} \leqslant t \leqslant T_{y} \end{cases}$$
(35)

• Condition 4:

The expected travel-between time $E_4(TB_2)$ for DC corresponding to the 4th condition becomes:

$$E_4(\text{TB}_2) = \int_{A(i,j)}^{T_y} t \cdot f_4(t) dt$$
(36)

• The expected travel-between time

The expected travel-between time E(TB) for DC for two randomly selected points corresponding to the travelling and transferring in the horizontal and moving in the vertical direction becomes:

$$E(TB) = \frac{1}{M^2} \left[\sum_{i=1}^{M} \sum_{\substack{j=1\\i\neq j}}^{M} E_{k(i,j)}(TB_2) + M \cdot E_k(TB_1) \right]$$

$$E_{k(i,j)} = \begin{cases} E_{1(i,j)}, \text{ if the condition 1 is fulfilled} \\ E_{2(i,j)}, \text{ if the condition 2 is fulfilled} \\ E_{3(i,j)}, \text{ if the condition 3 is fulfilled} \\ E_{4(i,j)}, \text{ if the condition 4 is fulfilled} \end{cases}$$
(37)

3.4.4. Expected dual command travel time for multi-aisle AS/RS when $i \neq j$

According to movement of the hoisted carriage in the vertical direction and travelling and transferring of the S/R machine in the horizontal direction, the expected dual command travel time for multi-aisle AS/RS corresponding to the *k*th condition and the condition $i \neq j$ is represented with the following expression:

$$E_k(DC)_{MASS} = E_k(SC)_{MASS} + E(TB)$$
(38)

4. Simulation model of multi-aisle AS/RS

To facilitate the evaluation of the performance and comparison of the proposed analytical travel time models for multi-aisle AS/RS, the discrete event simulation has been employed Lerher (2005). The simulation model of multi-aisle AS/RS consists of a single cross warehouse aisle and picking aisles, two lines of SR between picking aisle, S/R machine with the traverser (transfer car), I/O locations, and has been built using computer software *AutoMod* version 12.1 (Applied Materials, AutoMod 2009a).

The logistics of the simulation model is represented with the process which marks all storage compartments in the warehouse according to the prescribed storage area. After creating the list of free storage locations, the first TUL, which is situated in the I/O_{war} location, enters the simulation model. Further on, the TUL receives a sign which belongs to the *i*th pick aisle. The S/R machine picks up the TUL from the I/O_{war} location, loads it into the hoisted carriage, and transfers it through the cross warehouse aisle to the *i*th picking

aisle. After conducting the transfer to the *i*th picking aisle, the S/R machine moves from the I/O_{aisle(i)} location to the storage location simultaneously in the horizontal and vertical directions. For the storage operation, the randomized storage policy has been used. Next, the TUL that has been stored is put on the waiting list by a computer (computer data base), where it waits for the retrieval operation. For the retrieval process the first-come-first-serve (FCFS) request selection rules have been used. After the storage operation in the *i*th pick aisle, the S/R machine travels to the retrieval location in *j*th pick aisle. The retrieval location can be positioned in the same aisle (i = j) or in the adjacent aisle and nonadjacent aisles $(i \neq j)$. Next, the S/R machine loads TUL into the hoisted carriage and moves through the pick and cross warehouse aisle to the I/Owar location, where the TUL departs the system. The mean dual command travel time $T(DC)_{MASS}$ is therefore associated with the transferring of the S/R machine through the cross warehouse aisle and travelling in the pick aisle. As a performance measure of simulation analysis, the mean dual command travel time of multi-aisle AS/RS has been used (Applied Materials, AutoStat 2009b).

5. Multi-aisle AS/RS (case study)

For the performance comparison of the proposed analytical models with the simulation (discrete) model, we originate from the next parameters of multi-aisle AS/RS: $w = 0.8 \text{ m}, h = 0.8 \text{ m}, g = 1.2 \text{ m}, n = 3, b_1 = 0.1 \text{ m}, b_2 = 0.2 \text{ m}, b_3 = 0.12 \text{ m}, b_4 = 0.162 \text{ m}, b_5 = 0.3 \text{ m}, b_6 = 0.2 \text{ m}, s_x = 1.5 \text{ m}, W_{aisle} = 1.5 \text{ m}$. The length of the storage compartment equals next expression: $l_{cont} = b_2 + (w, n) + (n + 1)b_1$.

$$l_{\rm COM} = 2.920 \text{ m}$$
 (39)

The height of the storage compartment equals next expression: $h_{\text{COM}} = b_4 + h + b_2$ $h_{\text{COM}} = 1.162 \text{ m}$ (40)

In order to receive the best representative results, three different multi-aisle AS/RS with pick aisles M = (1, ..., 5) and one cross warehouse aisle have been used in our analyses:

- multi-aisle AS/RS I (
$$N_x = 10$$
 and $N_y = 5$)

- multi-aisle AS/RS II ($N_x = 20$ and $N_y = 11$),
- multi-aisle AS/RS III ($N_x = 27$ and $N_y = 17$).

According to the efficiency of the S/R machine, Stöklin automatic aisle transferring S/R machine for serving multiple pick aisles has been used.

5.1. Analyses and evaluation of results

The expected and mean dual command travel times in seconds for the multi-aisle AS/RS, which are presented in the following Table 3, are given on the basis of the performed analyses. Analyses have been conducted for three different multi-aisle AS/RS (see Table 1) with pick aisles M = 1, ..., 5 and one cross warehouse aisle, under the condition that the storage and retrieval request can occur: in the *i*th pick aisle – application of the condition i = j and in the *i*th and *j*th pick aisle – application of the condition $i \neq j$. In order to receive the best representative average of the dual command travel time, the simulation results presented in Tables 2 and 3 correspond to hundred thousand runs for every single type of the multi-aisle AS/RS.

According to the simulation results presented in Table 3, the performance comparison between the proposed model and model from Hwang and Ko (1988) has been calculated with the next expression:

$$\lambda = \frac{(E(DC)_{MASS} \cdot 100)}{T(DC)_{MASS}} - 100$$
(41)

According to the condition i = j we can notice that deviations of $E(DC)_{MASS}$ of the proposed models first increases (up to 4% in case M = 2) and are next reduced and drawn near the zero in case of large number of pick aisles M (Fig. 6). When taking into consideration the condition $i \neq j$ a changed deviational characteristic of $E(DC)_{MASS}$ can be noticed. The diagram in Fig. 7 shows an increase of deviations of $E(DC)_{MASS}$ (up to 5%) in dependence on the increase of the number of pick aisles *M*. This dependence can be explained with the fact that when deriving the expressions related to the condition $i \neq j$, the velocity-time characteristic II (see Eq. (4)) was taken into account. Also the deviation is the result of the application of the continuous models, while the simulation is based on the random recurrence of discrete events. With a small number of pick aisles M the mentioned characteristic is less obvious, but the deviations of $E(DC)_{MASS}$ are stabilized with the increase of the number of pick aisles M. It is generally valid that when ensuring the condition $i \neq j, E(DC)_{MASS}$ is higher for a part of time which is needed by the S/R machines for the transfer between optional pick aisles *i* and *j* (Table 3). The efficiency of the multi-aisle AS/RS will then get bigger if the condition $i \neq j$ in contrast to the condition i = j is represented to a much lesser extent, which is also dependent on the warehouse management system.

The analytical models by Hwang and Ko (1988) are based on the assumption that the velocity-time dependence of the S/R machine is considered only with the application of the uniform velocity. The deficiency of the existent analytical model is noticed with all three types of multi-aisle AS/RS. On the basis of the performed analysis it can be presumed that analytical models by Hwang and Ko (1988) approach the real situation only in the following cases:

- (a) When the length of the storage rack is very long (large L) and the influence of acceleration and deceleration is negligible with regard to the travelling of the S/R machine with constant velocity. In this way the part of time with constant velocity comes out, while the parts of time for acceleration and deceleration are negligibly small.
- (b) When the following conditions are fulfilled:

acceleration
$$\Rightarrow \infty$$
 $t_{acceleration} \Rightarrow 0$
deceleration $\Rightarrow \infty$ $t_{deceleration} \Rightarrow 0$ (42)

Table 1Multi-aisle AS/RS under study.

	<i>L</i> (m)	<i>H</i> (m)	<i>W</i> _{<i>M</i>=1} (m)	$W_{M=2}$ (m)	<i>W</i> _{<i>M</i>=3} (m)	$W_{M=4}$ (m)	<i>W_{M=5}</i> (m)	
multi-aisle AS/RS I	30.82	6.11	3.9	8	12.1	16.2	20.3	
multi-aisle AS/RS II	60.020	13.082	3.9	8	12.1	16.2	20.3	
multi-aisle AS/RS III	80.460	20.054	3.9	8	12.1	16.2	20.3	

Table	2	
abic	~	

Automatic aisle transferring S/R machine.

S/R machine	Horizontal direction	Vertical direction
The velocity (m/s) The acceleration/deceleration (m/s ²) Automatic transfer car "traverser"	3 0.5 Hori	0.8 0.8 zontal direction
The velocity (m/s) The acceleration/deceleration (m/s ²)	0.6 0.4	

Table 3	
Comparison of the expected and mean dual command travel times for the selected type of multi-aisle AS/RS.	

	Condition $(i = j)$			Condition $(i \neq j)$			
	Simulation	$v = \text{const.}$ Proposed models $v \neq \text{ const.}$		Simulation $v = \text{const.}$		Proposed models $v \neq \text{const.}$	
	$T(DC)_{MASS}(s)$	$E(DC)_{MASS}$ (s)	$E(DC)_{MASS}(s)$	$T(DC)_{MASS}$ (s)	$E(DC)_{MASS}$ (s)	$E(DC)_{MASS}$ (s)	
Multi-aisle AS/RS I							
<i>M</i> = 1	$30.02^{a}(0.00)$	15.87 ^b (-47.14)	30.18 ^c (0.53)	$30.02^{a}(0.00)$	15.87 ^b (-47.14)	30.18 ^c (0.53)	
<i>M</i> = 2	38.36 (0.00)	21.71 (-43.40)	39.9 (4.01)	49.18 (0.00)	27.97 (-43.13)	47.75 (-2.90)	
<i>M</i> = 3	45.7 (0.00)	28.22 (-38.25)	46.79 (2.39)	61.66 (0.00)	38.07 (-38.26)	58.78 (-4.67)	
M = 4	52.79 (0.00)	34.88 (-33.93)	53.64 (1.61)	72.39 (0.00)	47.68 (-34.13)	68.84(-4.90)	
<i>M</i> = 5	59.79 (0.00)	41.62 (-30.39)	60.48 (1.15)	82.51 (0.00)	57.12 (-30.77)	78.51 (-4.85)	
Multi-aisle	e AS/RS II						
<i>M</i> = 1	45.65 (0.00)	32.48 (-28.85)	46.2 (1.20)	45.65 (0.00)	32.48 (-28.85)	46.2 (1.20)	
<i>M</i> = 2	53.97 (0.00)	37.48 (-30.55)	55.93 (3.63)	68.36 (0.00)	46.45 (-32.05)	66.5 (-2.72)	
<i>M</i> = 3	61.31 (0.00)	43.4 (-29.21)	62.81 (2.45)	81.97 (0.00)	56.88 (-30.61)	78.44 (-4.31)	
M = 4	68.44 (0.00)	49.78 (-27.26)	69.66 (1.78)	93.42 (0.00)	66.68 (-28.62)	88.95 (-4.78)	
<i>M</i> = 5	75.38 (0.00)	56.34 (-25.26)	76.50 (1.49)	103.84 (0.00)	76.18 (-26.64)	98.9 (-4.76)	
Multi-aisle AS/RS III							
<i>M</i> = 1	58.12 (0.00)	46.28 (-20.37)	58.98 (1.48)	58.12 (0.00)	46.28 (-20.37)	58.98 (1.48)	
<i>M</i> = 2	66.47 (0.00)	50.67 (-23.77)	68.71 (3.37)	83.79 (0.00)	61.29 (-26.85)	81.06 (-3.26)	
<i>M</i> = 3	73.83 (0.00)	55.92 (-24.26)	75.60 (2.40)	98.41 (0.00)	71.58 (-27.26)	93.57 (-4.92)	
M = 4	80.94 (0.00)	61.78 (-23.67)	82.45 (1.87)	110.33 (0.00)	81.1 (-26.49)	104.37 (-5.40)	
<i>M</i> = 5	87.89 (0.00)	68 (-22.63)	89.29 (1.59)	120.99 (0.00)	90.41 (-25.27)	114.49 (-5.37)	

^{a,b,c} In case M = 1, the storage and the retrieval request is performed in the same picked aisle only, therefore travel times under the condition i = j equal travel times under the condition $i \neq j$.



Fig. 6. The performance comparison of the proposed models with the condition i = j.



Fig. 7. The performance comparison of the proposed models with the condition $i \neq j$.

If there were "theoretically" highly efficient engines, the values of acceleration and deceleration would approach infinity. Consequently, in this case the times for acceleration and deceleration come near the zero and thus the travelling of the S/R machine with constant velocity v_{max} starts.

(c) If the times of loading and unloading TUL, times of input and output identification are high. In this case they prevail throughout the whole time of the command cycle and this is why the travel time has a relatively small influence on the value of the whole time of the command cycle.

6. Conclusion

In this paper, performance improvements utilizing proposed analytical travel time models for the multi-aisle AS/RS are presented. According to other researchers Bozer and White (1984). Rosenblatt and Roll (1993), Hausman et al. (1976), Hwang and Ko (1988); who have in their analytical travel time models used mean uniform velocity only, the real operating characteristics of the S/R machine have been used in the proposed models. In proposed models we originate from travelling of the S/R machine in the pick aisle and transferring in the cross warehouse aisle. Thus, considering both movements of the S/R machine in the pick and in cross warehouse aisles, the proposed analytical travel time models have been developed. The proposed models deal also with cases when the sequencing of storage and retrieval request is performed in two randomly chosen pick aisles. Various elements of the multiaisle AS/RS have been examined, such as the layout of the SR and the efficiency of the S/R machine, in order to investigate the efficiency of the proposed models in comparison with the simulation model of multi-aisle AS/RS.

According to the analytical models which are based on constant velocity only, the largest deviation of $E(DC)_{MASS}$ occurs in range from 30% to 48%. Therefore the evaluated $E(DC)_{MASS}$ is under evaluated, which indicates the difficulty in planning the multi-aisle AS/RS. On the contrary, the proposed analytical travel time models show satisfactory deviations of $E(DC)_{MASS}$ (max 5.4%). The mentioned dependency refers to all three types of multi-aisle AS/RS and the conditions (i = j) and ($i \neq j$). Therefore the proposed analytical travel time models demonstrate good performances for the multi-aisle AS/RS. The proposed models could be of considerable help to professionals in practice, when making decisions in the early stages of design project of multi-aisle AS/RS and when deciding which type of S/R machine will be most promising.

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Appendix A. Verification of expressions 2 and 4

• Verification of the expression 2 where the peak velocity v_p is less than v_{max} .

The velocity in dependence of time v(t) equals the following expression:

$$v(t) = \begin{cases} at, & t \in (0, t_p) \\ -a(t-T), & t \in (t_p, T) \end{cases}$$
(43)

Distances in dependence of time $d_1(T)$ and $d_2(T)$ according to conditions 1 and 2 equal the following expressions (see Fig. 2):

• Condition 1:
$$0 \leq t \leq t_p$$

$$a(t) = a$$

$$v(t) = a \cdot t$$

$$d_1(t) = \int_0^{t_p} v(t)dt = \int_0^{t_p} a \cdot tdt = a \cdot \frac{t_p^2}{2}$$
(44)

• Condition 2: $t_p \leq t \leq T$

$$a(t) = a$$

$$v(t) = -a(t - T)$$

$$d_{2}(t) = \int_{t_{p}}^{T} v(t)dt = \int_{t_{p}}^{T} -a(t - T)dt = \frac{1}{2}a(T - t_{p})^{2}$$
(45)

The distance in dependence of time d(T) equals the following expression:

$$d(T) = d_1(t) + d_2(t) = a \cdot \frac{t_p^2}{2} + \frac{1}{2}a(T - t_p)^2$$
(46)

Considering the condition $t_p = T/2$, the distance in dependence of time d(T) equals the next expression:

$$d(T) = \frac{a \cdot T^2}{4} \tag{47}$$

• Verification of the expression 4 where the peak velocity v_p is equal to v_{max} .

The velocity in dependence of time v(t) equals the following expression:

$$v(t) = \begin{cases} at, & t \in (0, t_p) \\ v_{max}, & t \in (t_p, T - t_p) \\ -a(t - T), & t \in (T - t_p, T) \end{cases}$$
(48)

Distances in dependence of time $d_1(T)$, $d_2(T)$ and $d_3(T)$ according to conditions 1, 2 and 3 equal the following expressions (see Fig. 2):

• Condition 1:
$$0 \leq t \leq t_p$$

$$a(t) = a$$

$$v(t) = a \cdot t$$

$$d_1(t) = \int_0^{t_p} v(t)dt = \int_0^{t_p} a \cdot tdt = \frac{at_p^2}{2}$$

• Condition 2: $t_p \leq t \leq T - t_p$

$$a(t) = 0$$

$$v(t) = v_{max}$$

$$d_{2}(t) = \int_{t_{p}}^{T-t_{p}} v(t)dt = \int_{t_{p}}^{T-t_{p}} v_{max}dt = v_{max}(T-2t_{p})$$
(50)

• Condition 3: $T - t_p \leq t \leq T$

$$a(t) = a$$

$$v(t) = -a(T - t_p, T)$$

$$d_2(t) = \int_{T - t_p}^{T} v(t)dt = \int_{T - t_p}^{T} -a(T - t)dt = \frac{at_p^2}{2}$$
(51)

The distance in dependence of time d(T) equals the following expression:

$$d(T) = d_1(t) + d_2(t) + d_3(t) = \frac{at_p^2}{2} + v_{max}(T - 2t_p) + \frac{at_p^2}{2}$$
(52)

Considering the condition $t_p = v_{max}/a$, the distance in dependence of time d(T) equals the next expression:

$$d(T) = v_{max} \cdot T - \frac{v_{max}^2}{a}$$
(53)

References

(49)

Applied Materials, 2009a. AutoMod 12.3, USA.

- Applied Materials, 2009b. AutoStat 4.4, USA.
- Bozer, Y.A., White, A.J., 1984. Travel-time models for automated storage and retrieval systems. IIE Transactions 16 (4), 329–338.
- De Koster, M.B.M., Le-Duc, T., Yu, Y., 2008. Optimal storage rack design for a 3dimensional compact AS/RS. International Journal of Production Research 46 (6), 1495–1514.
- Gnedenko, B.V., 1969. The Theory of Probability. MIR Publisher, Moscow.
- Graves, S.C., Hausman, W.H., Schwarz, L.B., 1977. Storage retrieval interleaving in automatic warehousing systems. Management Science 23 (9), 935–945.
- Gudehus, T., 1973. Principles of Order Picking: Operations in Distribution and Warehousing Systems. W. Girardet Verlag, Essen, Germany.
- Gu, J., Goetschalckx, M., McGinnis, L.F., 2007. Research on warehouse operation: a comprehensive review. European Journal of Operational Research 177, 1–21.
- Hausman, H.W., Schwarz, B.L., Graves, C.S., 1976. Optimal storage assignment in automatic warehousing systems. Management Science 22 (6), 629–638.
- Hwang, H., Lee, S.B., 1990. Travel time models considering the operating characteristics of the storage and retrieval machine. International Journal of Production Research 28 (10), 1779–1789.
- Hwang, H., Ko, C.S., 1988. A study on multi-aisle system served by a single storage and retrieval machine. International Journal of Production Research 26 (11), 1727–1737.
- Lerher, T., 2005. Model for designing Automated Storage and Retrieval Systems. Thesis (Ph.D.). University of Maribor, Faculty of Mechanical Engineering, Slovenia.
- Lerher, T., Šraml, M., Kramberger, J., Borovinšek, M., Zmazek, B., Potrč, I., 2005. Analytical travel time models for multi-aisle automated storage and retrieval systems. International Journal of Advance Manufacturing Technology 30 (3–4), 340–356.
- Lerher, T., 2006. Design and evaluation of the class-based multi-aisle AS/RS. International Journal of Simulation Modelling 5 (1), 25–37.
- Ramu, N.V., 1996. Design methodology for modelling warehouse internal layout integrated with operating policies. Thesis (Ph.D.), Clemson University, USA. Roodbergen, K.J., Vis, F.A., 2009. A survey of the literature on automated storage and
- retrieval systems. European Journal of Operational Research 194, 343–362.
- Rosenblatt, M.J., Roll, J., 1993. A combined optimization and simulation approach for designing automated storage and retrieval systems. IIE Transactions 25 (1), 40–50.
- Rushton, A., Croucher, P., Baker, P., 2006. Logistics and Distribution Management. The Chartered Institute of Logistics and Transport, UK.
- Sari, Z., Saygin, C., Ghouali, N., 2005. Travel-time models for flow-rack automated storage and retrieval systems. International Journal of Advanced Manufacturing Technology 25 (9–10), 979–987.
- Tollazzi, T., 2006. Transport and manipulation surfaces for trucks on highways, road terminals and logistics centers. Chapter of Scientific Research Work for Company DARS, Slovenia.
- Tompkins, J.A., White, J.A., Bozer, Y.A., Tanchoco, J.M.A., 2003. Facilities Planning. third ed.. John Wiley and Sons, New York, USA.

- Vössner, S., 1994. Spielzeit Berechnung von Regalförderzeugen. Thesis (Ph.D.). Graz University of Technology, Austria.
- Vidovics, H., 1994. Die Systemanalyse und Umschlagleistungen von Regalförderzeugen mit Mehrfachlastaufnahmemitteln, Thesis (Ph.D.). Graz University of Technology, Austria.
- Wen, U.P., Chang, D.T., Chen, S.P., 2001. The impact of acceleration and deceleration on travel time models in class-based automated S/R systems. IIE Transactions 33 (4), 599–608.
- Yu, Y., De Koster, M.B.M., 2009a. Designing an optimal turnover-based storage rack for a 3D compact AS/RS. International Journal of Production Research 47 (6), 1551–1571.
- Yu, Y., De Koster, M.B.M., 2009b. Optimal zone boundaries for two class-based compact 3D AS/RS. IIE Transactions 41 (3), 194–208.
- Zollinger, H., 1999. AS/RS application, benefits and justification in comparison to other storage methods: A white paper. Automated storage and retrieval systems production section of the Material handling industry of America.