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Travel time models for double-deep automated storage and retrieval systems

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In this paper new analytical travel time models for the computation of cycle times for unit-load double-deep automated storage and retrieval systems (in continuation double-deep AS/RS) are presented. The proposed models consider the real operating characteristics of the storage and retrieval machine and the condition of rearranging blocking loads to the nearest free storage location during the retrieval process. With the assumption of the uniform distributed storage rack positions and the probability theory, the expressions of the single and modified dual command cycle have been determined. The proposed models enable the calculation of the mean cycle time for single and dual command cycles, from which the performance of the double-deep AS/RS can be evaluated. A simulation model of the selected double-deep AS/RS has been developed to compare the performances of the proposed analytical travel time models. The numerical analyses show that with regard to the examined type of double-deep AS/RS with a different fill-grade factor, the results of the proposed analytical travel time models correlate with the results of simulation models of double-deep AS/RS.

Keywords: double-deep AS/RS; travel time analysis; mathematical modelling

1. Introduction

Warehouses are an important aspect of economic activity. With regard to the rapid development of technology in the material handling equipment and the increase in the cost of labour, new approaches in designing warehouses have been developed. Modern warehousing systems can be generally classified as mechanised (conventional) or automated warehouses. A combination of labour and material handling equipment is utilised in mechanised systems to facilitate receiving, storage and shipping processes. Generally, the labour constitutes a high percentage of the overall cost in mechanised systems. On the contrary, automated warehouses attempt to minimise the labour element as much as possible by making the capital investment in equipment (Ashayeri *et al.* 1985).

An important part of automated warehouses is represented by automated storage and retrieval systems (AS/RS). The basic components of AS/RS are storage racks (SR), storage and retrieval machines (S/R machines), input/output (I/O) locations and accumulating conveyors. AS/RS offers the advantages of fewer material handlers, better material control (including security), and more efficient use of storage space. The disadvantage is that the

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capital and maintenance costs are high, and it is difficult to modify such a system once it is installed. As a result of this, the adoption of AS/RS must be economically and operationally justifiable, and its deployment must be carefully planned. Essentially, AS/RS are designed to achieve a high throughput capacity and to support accurate picking of transport unit loads (TUL). A modern warehouse using AS/RS combines the state-of-the-art 'hardware' and information systems to achieve high-value added warehousing operations for a high variety of loads.

Various designs and configurations of the unit-load AS/RS have been made due to different requirements and operating conditions. Double-deep AS/RS essentially consist of two single-deep SR placed one behind the other, and so TUL are stored in the first or second storage lane of the SR. Each storage lane of the SR is independently accessible, and so any TUL can be stored in any storage lane at any level of the SR (Bartholdi and Hackman 2007). A typical double-deep AS/RS has the special design of the S/R machine, which moves the TUL to or from the first or second storage lane of the SR on the either side of the picking aisle. High lifting distances, high velocity in horizontal and vertical directions, precise handling and controlled movement are important features of the S/R machine. The S/R machine consists of a travelling structural frame, which is guided and supports a hoisted carriage on which the TUL is placed. The shuttle or load attachments on the hoisted carriage manipulate the TUL into and out of the SR position. Since TUL are stored in the first and second storage lane, a special design with double-deep load attachments, or with the satellite vehicle for heavier loads, is utilised. The advantage of the double-deep SR is that fewer aisles are needed, which results in a more efficient use of floor space. In general a 50% aisle space saving is achieved in comparison with the single-deep SR.

Many producers of the warehouse equipment, such as Siemens Dematic, Stöcklin, Mias Maschinenbau München, etc., have begun to offer double-deep and deep lane AS/RS. The selection of single- or double-deep systems depends on the required throughput capacity and on the desired level of flexibility in the warehouse.

The unit-load AS/RS have been the subject of many researchers over the past few years. Hausman *et al.* (1976) and Graves *et al.* (1977) have presented travel time models for unit-load AS/RS assuming that the SR is square-in-time (SIT). They have analysed different storage strategies, e.g. randomised, turnover-based and class-based storage assignment rules. Gudehus (1973) has presented basic principles for the determination of cycle times according to unit-load AS/RS. With regard to other cycle time expressions, he has considered the impact of the acceleration and deceleration rate on travel times. Bozer and White (1984) have presented analytical models for the calculation of single command cycle (SC) and dual command cycle (DC) for non-SIT racks. Their models are based on randomised storage and retrieval with different I/O configurations of the input queue. Hwang and Lee (1990) and Vössner (1994) have presented travel time models considering the operating characteristics of the S/R machine for unit-load AS/RS and non-SIT racks. While the majority of researchers have considered the unit-load single-deep AS/RS only, there is a lack of research for the unit-load double-deep AS/RS. Studies of double-deep AS/RS have been presented by authors Oser *et al.* (1998, 2004) and Ritonja (2003). Their study is based on travel time models considering the multi-shuttle and class based storage system for mini-load AS/RS. Studies of multi-deep AS/RS have been presented by the following authors. Sari *et al.* (2005) have presented the travel-time models for the 3D flow-rack AS/RS. They have introduced the flow-rack, where TUL loaded by the S/R machine by one end of the rack, travels to another end of the rack to be retrieved. For the storage operation, the S/R machine operates in the same way as the S/R machine in the unit-load AS/RS. However, the retrieval operation for a particular TUL requires that the S/R machine removes all TUL stored in front of the requested TUL. The acceleration/deceleration effect on the travel time was not considered by the authors. De Koster et al. (2008) presented an optimal storage rack design for the 3D compact AS/RS. They introduced the combination of the S/R machine for TUL movement in the horizontal and vertical directions and the system of inbound and outbound conveyors (powered or non-powered) for the depth movement. They analysed the system performance and optimal dimension of the system for the random storage strategy in order to minimise the expected travel time of the S/R machine. In the continuation of their work, Yu and De Koster (2008a) have derived the expected single command cycle time under the full-turnover based storage policy. In order to implement the class-based policy, Yu and De Koster (2008b) have introduced the model for determining the optimal storage zone boundaries for the 3D compact AS/RS. For the determination of the zone boundaries, they have introduced the mixed-integer nonlinear model. The presented research is a very useful and flexible tool when designing 3D compact AS/RS. According to the review of the multi-deep AS/RS the researchers used a different approach for analysing the system performance from the one that will be used in our paper.

According to the previous literature research, our paper presents the proposed analytical travel time models for unit-load double-deep AS/RS considering the real operating characteristics of the S/R machine. Since we are dealing with two storage lanes in the storage rack the blocking of the TUL during the retrieval process will be introduced. The proposed analytical travel time models are based on randomised storage and retrieval of TUL under the condition of rearrangement of blocking TUL during the retrieval process (the application of the FEM 9.851 guideline). FEM 9.851 is a commonly used procedure for calculating cycle times. The proposed analytical travel time models for unit-load double-deep AS/RS are also validated through discrete event simulations.

2. Statistical analysis of travel time

When developing the proposed analytical travel time models for double-deep AS/RS, the real operating characteristics of the S/R machine have been considered (Lerher et al. 2006). We estimate that the analytical travel time models based on the assumption that the S/R machine travels with a uniform velocity only are too general. If we limit ourselves to using the uniform velocity only, the average travel time for a cycle becomes shorter and consequently the throughput capacities become larger, which is not in accordance with the practice. The above-mentioned statement becomes even more evident by the short SR.

When developing the proposed analytical travel time models for double-deep AS/RS, the following assumptions were considered:

- The double-deep AS/RS is considered to be divided into picking aisles with double-deep SR on both sides of the aisle.
- The double-deep SR is considered to be a continuous rectangular pick face where the I/O location is located in the lower left-hand corner of the SR.
- The S/R machine operates either on single or dual command cycle. The hoisted carriage has place for only one TUL. In principle TUL will be stored first in the second storage lane of the SR, which will minimise the number of re-arrangements during the retrieval operation.

- The S/R machine is equipped with the load attachments and can serve in the first storage lane of the SR and in the second storage lane of the SR.
- The specification of the S/R machine (max. velocities in the horizontal and vertical directions and acceleration and deceleration rates) as well as the length and the height of the SR are known.
- The S/R machine and the hoisted carriage travel simultaneously in the horizontal and vertical directions in the aisle.
- The length and the height of the SR are long enough for the S/R machine to reach maximal velocity from the I/O location.
- The randomised storage assignment rule has been used.

The following notation is introduced:

v_x	Maximum velocity of the S/R machine in the horizontal direction.
v_y	Maximum velocity of the hoisted carriage in the vertical direction.
v_t	Telescopic load attachments velocity.
a_x^+	Acceleration of the S/R machine in the horizontal direction.
a_y^+	Acceleration of the hoisted carriage in the vertical direction.
a_x^-	Deceleration of the S/R machine in the horizontal direction.
a_y^-	Deceleration of the hoisted carriage in the vertical direction.
L	Length of the SR.
H	Height of the SR.
Q	SR capacity.
λ	Throughput capacity.
n_x	Number of SR locations in the horizontal direction.
n_y	Number of SR locations in the vertical direction.
n_{stored}	Number of occupied storage locations.
F	Cumulative distribution function.
E	Expected value.
P	Probability.
α	Fill-grade factor.
t	Variable travel time.
$T(\text{SC})$	Single command cycle time.
$T(\text{DC})_M$	Modified dual command cycle time.
t_c	Control time.
t_1	Pickup/setdown time in the first storage lane of the SR.
t_2	Pickup/setdown time in the second storage lane of the SR.
b_w	Bay width.
b_d	Bay depth.
a_w	Aisle width.

2.1 *Travelling of the S/R machine in the storage rack*

Travel times of the S/R machine to any randomly selected location in the SR are variable times. They depend on the kinematics properties of the S/R machine and the hoisted carriage, the length L and the height H of the S/R and the storage policy.

The travel time t_{xy} from the I/O location to any randomly selected location in the SR is the maximal value of t_x or t_y, where t_x is the horizontal travel time and t_y is the vertical travel time. According to the condition of uniform distribution of storage locations in the SR and to the condition of the x- and y-coordinates independence, the cumulative distribution functions $F_x(t)$ and $F_y(t)$ have been accomplished. The cumulative distributions functions $F_x(t)$ and $F_y(t)$ are distinguished according to the following condition:

(i) S/R machine travelling/hoisted carriage movement for type I (the obtained peak velocity is less than $v_{\max}$).
(ii) S/R machine travelling/hoisted carriage movement for type II (the obtained peak velocity is equal to $v_{\max}$).

2.1.1 *Travelling of the S/R machine and movement of the hoisted carriage from the I/O point to a randomly selected location in the SR*

- S/R machine travelling in the horizontal direction (Hwang and Lee 1990)

$$F_x(t) = \begin{cases} \frac{a_x t^2}{4L} \left(0 \le t \le \frac{2v_x}{a_x}\right) \\ \frac{v_x t}{L} - \frac{v_x^2}{a_x L} \left(\frac{2v_x}{a_x} \le t \le \frac{L}{v_x} + \frac{v_x}{a_x}\right) \end{cases}. \tag{1}$$

- Hoisted carriage movement in the vertical direction (Hwang and Lee 1990)

$$F_y(t) = \begin{cases} \frac{a_y t^2}{4H} \left(0 \le t \le \frac{2v_y}{a_y}\right) \\ \frac{v_y t}{H} - \frac{v_y^2}{a_y H} \left(\frac{2v_y}{a_y} \le t \le \frac{H}{v_y} + \frac{v_y}{a_y}\right) \end{cases}. \tag{2}$$

2.1.2 *Travelling of the S/R machine and movement of the hoisted carriage from a storage location to a randomly selected retrieval location in the SR*

- S/R machine travelling in the horizontal direction (Hwang and Lee 1990)

$$F_x(t) = \begin{cases} \frac{a_x}{2L} t^2 - \frac{a_x^2}{16L^2} t^4 \left(0 \le t \le \frac{2v_x}{a_x}\right) \\ -\frac{v_x^2}{L^2} t^2 + \left[\frac{2v_x^3}{a_x L^2} + \frac{2v_x}{L}\right] t - \frac{2v_x^2}{a_x L} - \frac{v_x^4}{a_x^2 L^2} \left(\frac{2v_x}{a_x} \le t \le \frac{L}{v_x} + \frac{v_x}{a_x}\right) \end{cases}. \tag{3}$$

- Hoisted carriage movement in the vertical direction (Hwang and Lee 1990)

$$F_y(t) = \begin{cases} \frac{a_y}{2H} t^2 - \frac{a_y^2}{16H^2} t^4 \left(0 \le t \le \frac{2v_y}{a_y}\right) \\ -\frac{v_y^2}{H^2} t^2 + \left[\frac{2v_y^3}{a_y H^2} + \frac{2v_y}{H}\right] t - \frac{2v_y^2}{a_y H} - \frac{v_y^4}{a_y^2 H^2} \left(\frac{2v_y}{a_y} \le t \le \frac{H}{v_y} + \frac{v_y}{a_y}\right) \end{cases}. \tag{4}$$

The distribution functions $F_{ik}(t)$, where ($i = 1$ or 2) and ($k = 1, \ldots, 6$), depend on the relationships among the values of the following parameters: v_x, v_y, a_x, a_y, L, H. Therefore $F_{ik}(t)$ can be specified with theoretically six different cases:

$$F_{ik}(t) = F_x(t) \cdot F_y(t) \quad \text{for } 0 \le t \le T, \tag{5}$$

where $F_{1k}(t)$ stands for travelling of the S/R machine and movement of the hoisted carriage from the I/O point to a randomly selected location in the SR and $F_{2k}(t)$ stands for travelling of the S/R machine and movement of the hoisted carriage from a storage location to a randomly selected retrieval location in the SR.

The expected travel time $E_{ik}(t)$, corresponding to the kth condition is equal to the next expression:

$$E_{ik}(t) = \int_0^{\max(T_x,T_y)} (1 - F_{ik}(t))\mathrm{d}t. \tag{6}$$

For a more detailed insight into the statistical analysis of travel time considering the real operating characteristics of the S/R machine, see the research of Vössner (1994), Hwang and Lee (1990), Lerher (2005) and Ritonja (2003).

3. Analytical analysis of travel time in the double-deep AS/RS

In the case of double-deep AS/RS, the average travel time for dual command cycle is enlarged for the rearrangement of the blocking TUL to the nearest free storage location in the SR. When developing the proposed analytical travel time models for double-deep AS/RS, the assumptions presented in FEM 9.851 guideline, have been considered. The FEM 9.851 guideline introduces the condition that the rearrangement of the blocking TUL can occur during the storage and retrieval processes. According to the presented storage strategy the storage process in the first or in the second storage lane, the TUL will always be first stored in the second storage lane. When all storage locations in the second storage lane are full, the storage in the first storage lane begins. According to the latter, the possibility of the rearrangement of the blocking TUL is eliminated. On the contrary, in the case of retrieval process the access to the second storage lane might be blocked by the blocking TUL in the first storage lane (a special case of retrieval). The problem can be solved by rearranging the blocking TUL to the nearest free storage location in the SR by using the Euclidian distance. The rearrangement of TUL is performed by the next sequence (see Figure 1): picking up the blocking TUL from the first storage lane of the SR (single-depth); repositioning the S/R machine to the nearest free storage location; setting down the formerly blocking TUL in the first or the second storage lane of the SR (single- or double-depth); repositioning the S/R machine to the earlier retrieval location; picking up the TUL to retrieve from the second storage lane of the SR (double-depth).

The rearranging of blocking TUL to the nearest free storage location is influenced by:

- Fill-grade factor α, which has an affect on the rearrangement distances and the number of rearrangements.
- Storage strategies.

3.1 *The definition of the rearrangement distance*

The average rearrangement distance and consequently the average travel time to the nearest free storage location in the SR, using the randomised storage assignment rule, is influenced by the fill-grade factor α. The average rearrangement distance

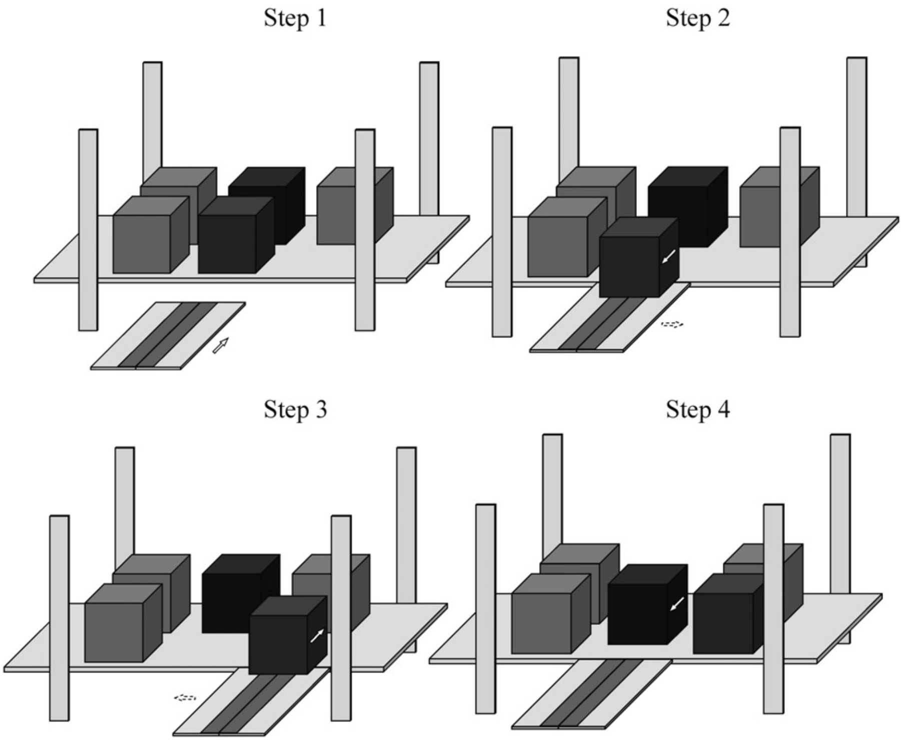

Figure 1. The procedure of rearranging TUL.

increases by increasing the fill-grade factor α. The travel time for the rearrangement represents only a small part of the cycle time, therefore it is possible to use the approximation for all storage locations in the SR. In the proposed analytical travel time models for the calculation of the average rearrangement distance, the FEM 9.851 guideline and the results of simulation modelling analysis (Ritonja 2003) have been used.

The expected value of the rearrangement distances E_x in the horizontal and E_y in the vertical direction is equal to the next expressions:

$$\begin{aligned} E_x &= \frac{1}{3} \cdot \frac{L}{n_x} \sqrt{\frac{1}{(1-\alpha)}} \\ E_y &= \frac{1}{3} \cdot \frac{H}{n_y} \sqrt{\frac{1}{(1-\alpha)}}. \end{aligned} \tag{7}$$

The fill-grade factor α is determined by the next relationship:

$$\alpha = \frac{n_{\text{stored TUL}}}{\left(n_x \cdot n_y\right)_{\text{total TUL}}}. \tag{8}$$

3.2 *The influence of the fill-grade factor α on the rearrangement of the TUL*

3.2.1 *The introduction of the storage strategy with the fill-grade factor α [0, 0.5]*

The storage strategy with the fill-grade factor α $[0, 0.5]$ (see Figure 2) takes into consideration that:

- the TUL will always be first stored in the second storage lane of the SR,
- when all storage locations in the second storage lane of the SR are full ($\alpha > 0.5$), the storage in the first storage lane of the SR begins,
- the possibility of the rearrangement of the blocking TUL is eliminated during the storage and the retrieval processes.

The second storage lane of the SR equals:

The probability that the storage location in double-deep AS/RS is occupied is equal to the next expression

$$P_{\text{Occ.}} = 2\alpha. \tag{9}$$

The probability that the storage location in double-deep AS/RS is empty is equal to the next expression

$$P_{\text{Free}} = 1 - 2\alpha. \tag{10}$$

The first storage lane of the SR equals:

The probability that the storage location in double-deep AS/RS is occupied is equal to the next expression:

$$P_{\text{Occ.}} = 0. \tag{11}$$

The probability that the storage location in double-deep AS/RS is empty is equal to the next expression:

$$P_{\text{Free}} = 1. \tag{12}$$

According to the value of the fill-grade factor α, which can be in the range from $[0, 0.5]$ two different cases of the storage and retrieval processes (see Figure 3) can occur during the working cycle (Ritonja 2003).

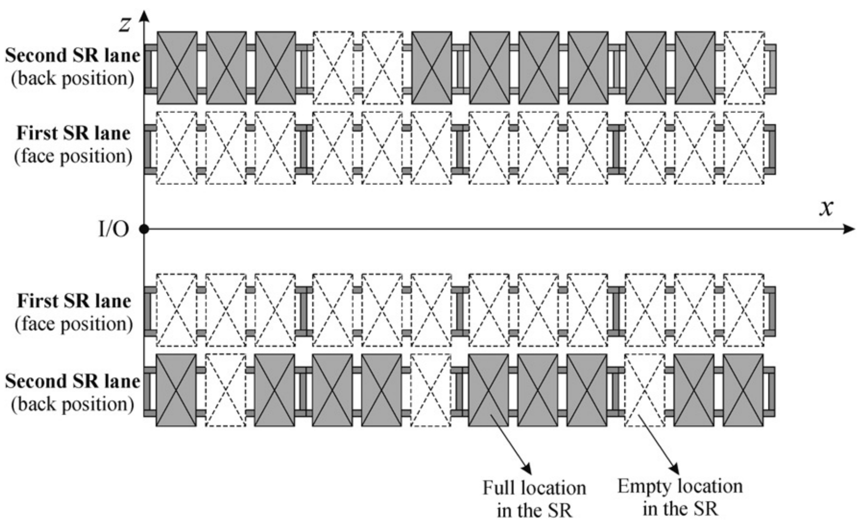

Figure 2. The layout of the double-deep SR with the fill-grade factor α $[0, 0.5]$.

Case 1 (the first and the second storage lanes of the SR are both free)**:** The probability P_1 that the storage locations in the first and in the second storage lane of the SR are empty is equal to the next expression:

$$P_1 = P_{\text{Free}} \cdot P_{\text{Free}} = 1 - 2\alpha. \tag{13}$$

Case 2 (the first storage lane of the SR is empty, while the second storage lane of the SR is occupied)**:** The probability P_2 that the storage locations in the first storage lane of the SR are empty and occupied in the second storage lane of the SR is equal to the next expression:

$$P_2 = P_{\text{Free}} \cdot P_{\text{Occ.}} = 2\alpha. \tag{14}$$

3.2.2 *The introduction of the storage strategy with the fill-grade factor α [0.5, 1]*

The storage strategy with the fill-grade factor α [0.5, 1] (see Figure 4) takes into consideration that

- because all storage locations in the second storage lane of the SR are full ($\alpha > 0.5$), the storage in the first storage lane of the SR begins,

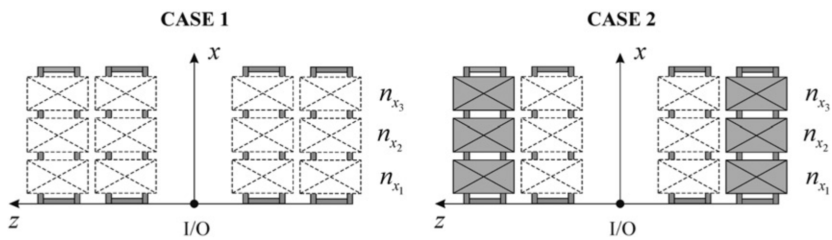

Figure 3. The two different cases of the storage and retrieval processes.

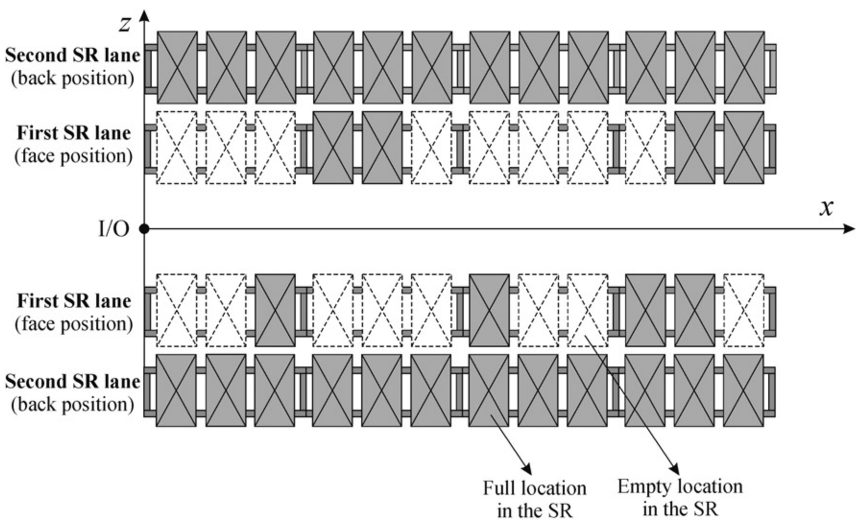

Figure 4. The layout of the double-deep SR with the fill-grade factor α [0.5, 1].

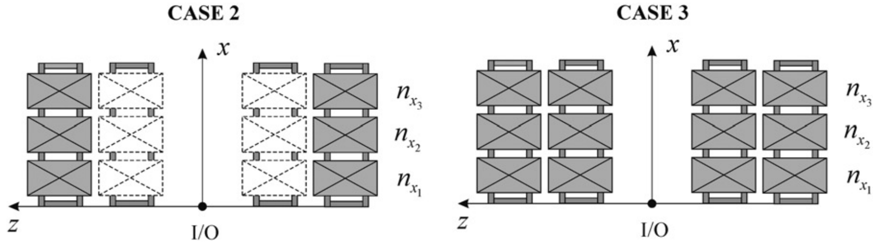

Figure 5. The two different cases of the storage and retrieval processes.

- the possibility of the rearrangement of the blocking TUL is eliminated during the storage process,
- because the second storage lane of the SR might be blocked by the blocking TUL in the first storage lane of the SR, the rearrangement of the blocking TUL to the nearest free storage location during the retrieval process is introduced.

The second storage lane of the SR equals:

The probability that the storage location in double-deep AS/RS is occupied is equal to the next expression:

$$P_{\text{Occ.}} = 1. \tag{15}$$

The probability that the storage location in double-deep AS/RS is empty is equal to the next expression:

$$P_{\text{Free}} = 0. \tag{16}$$

The first storage lane of the SR equals:

The probability that the storage location in double-deep AS/RS is occupied is equal to the next expression:

$$P_{\text{Occ.}} = 2\alpha - 1. \tag{17}$$

The probability that the storage location in double-deep AS/RS is empty is equal to the next expression:

$$P_{\text{Free}} = 2 - 2\alpha. \tag{18}$$

According to the value of the fill-grade factor α, which can be in the range from 0.5 to 1, two different cases of the storage and retrieval processes (see Figure 5) can occur during the working cycle (Ritonja 2003).

Case 2 (the first storage lane of the SR is empty, while the second storage lane of the SR is occupied)**:** The probability P_2 that the SR locations in the first storage lane of the SR are empty and occupied in the second storage lane of the SR is equal to the next expression:

$$P_2 = P_{\text{Free}} \cdot P_{\text{Occ.}} = 2 - 2\alpha. \tag{19}$$

Case 3 (the first and the second storage lanes of the SR are both occupied)**:** The probability P_3 that the SR locations in the first and in the second storage lanes of the SR are occupied is equal to the next expression:

$$P_3 = P_{\text{Occ.}} \cdot P_{\text{Occ.}} = 2\alpha - 1. \tag{20}$$

3.3 *The probability of rearrangements during the storage and retrieval processes*

3.3.1 *The fill-grade factor α [0, 0.5]*

According to our assumption in Section 3.2.1, the probability of rearrangement during the storage and the retrieval processes is equal to the following expression:

The storage process

$$P_{R\,(\text{storage})} = 0. \tag{21}$$

The retrieval process

$$P_{R(retrieval)} = 0. \tag{22}$$

Because of the assumption that the probability of the rearrangement during the storage and the retrieval processes is equal to zero, the probability P_R of the rearrangement during the dual command cycle equals 0.

3.3.2 *The fill-grade factor α [0.5, 1]*

According to our assumption in Section 3.2.2, the probability of rearrangement during the storage and the retrieval processes is equal to the following expression:

The storage process

$$P_{R\,(\text{storage})} = 0. \tag{23}$$

The retrieval process

During the retrieval process and the random or first come-first serve (FCFS) storage policy, the rearrangement of the blocking TUL (see Figure 6) will occur. It must be emphasised that the randomised storage assignment rule in the case of an infinite number of runs is equal to the FCFS storage policy.

The probability of the rearrangement during the retrieval process is expressed by the relationship of the probability for the blockade (Case 3) and the sum of all probabilities P_i of the retrieval locations.

$$P_{R\,(\text{retrival})} = \frac{P_3}{P_2 + 2P_3} = \frac{2\alpha - 1}{2 - 2\alpha + 2(2\alpha - 1)} = \frac{2\alpha - 1}{2\alpha}. \tag{24}$$

Because of the assumption that the probability of the rearrangement during the storage process is equal to zero, the probability of the rearrangement during the retrieval process is equal to the following expression:

$$P_R = \frac{P_{R\,(\text{storage})} + P_{R\,(\text{retrival})}}{2} = \frac{2\alpha - 1}{4\alpha}. \tag{25}$$

4. Cycle time in double-deep AS/RS when α [0, 0.5]

4.1 *Single command cycle (single storage or single retrieval in the cycle)*

4.1.1 *Single storage operation*

For a single storage operation (Figure 7), the single command cycle consists of the time of picking up the TUL at the pickup/deliver (PD) station, travelling to the destination

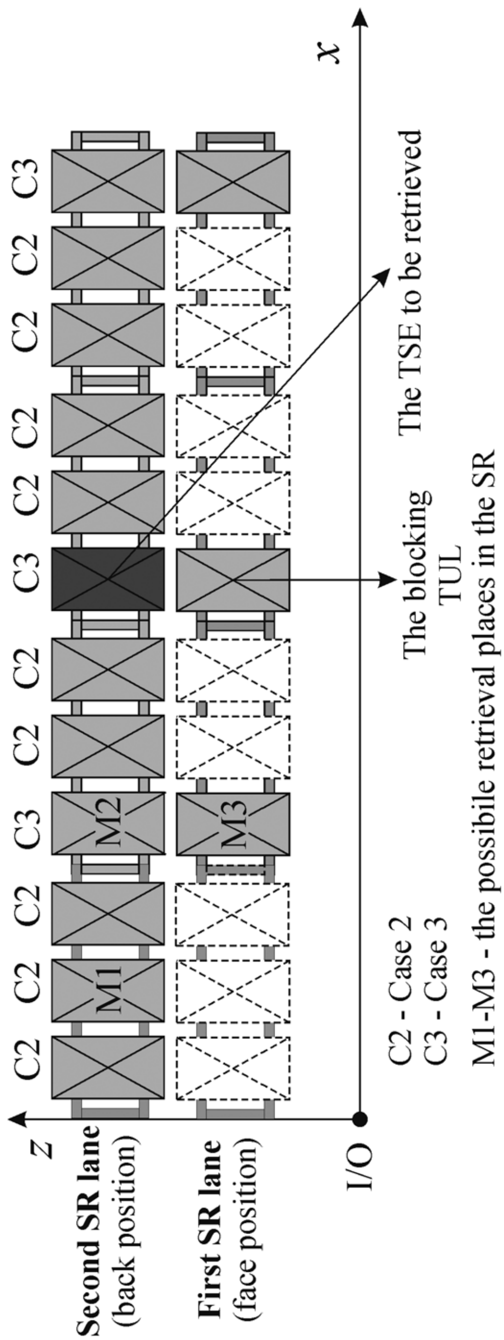

Figure 6. The rearrangement in the double-deep AS/RS.

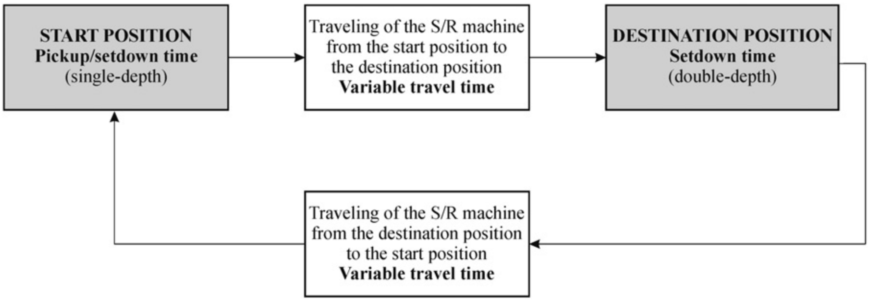

Figure 7. The course of single storage operation when λ $[0, 0.5]$.

position and setting down the TUL in the second storage lane of the SR, returning empty to the start position and setting down the TUL at the PD station. The cycle time is increased by the necessary control time, which stands for the waiting time of processing programs, sensor signals and the time of stabilisation of the S/R machine. The control time could be associated with the load handling time. Therefore it is not directly included in our proposed models.

The expected single command cycle time E (SC) equals the next expression:

$$E\,(\mathrm{SC}) = 2t_{PD} + 2 \cdot E\big(t_{p_i, p_{i+1}}\big) + t_2, \tag{26}$$

where t_{PD} is the pickup/setdown time at the PD station (sec.), $t_{p_i,\ p_{i+1}}$ is the travel time from the start position p_i to the destination position p_{i+1} (sec.), and t_2 is the pickup/setdown time in the second storage lane of the SR (sec.),

4.1.2 *Single retrieval operation*

The time of the single retrieval operation is developed inversely due to the storage operation. Because there is no rearrangement in the single command cycle the expected single command cycle time E (SC) in case of retrieval equals expression (26).

4.2 *Dual command cycle (single storage and single retrieval in the cycle)*

The dual command cycle (Figure 8) involves both storage and retrieval operations simultaneously. The dual command cycle time consists of the time of picking up the TUL at the PD station, travelling to the destination position and setting down the TUL in the second storage lane of the SR, travelling empty to the retrieval position, retrieving the TUL from the second storage lane, returning to the start position and setting down the TUL at the PD station.

The expected dual command cycle time E (DC) equals the next expression:

$$E\,(\mathrm{DC}) = 2 \cdot t_{PD} + 2 \cdot E\big(t_{p_i, p_{i+1}}\big) + 2 \cdot E\big(t_{p_{i+1}, p_{i+2}}\big) + 2t_2, \tag{27}$$

where t_{PD} is the pickup/setdown time at the PD station (sec.), $t_{p_i,\ p_{i+1}}$ is the travel time from the start position p_i to the destination position p_{i+1} (sec.), $t_{p_{i+1},\ p_{i+2}}$ is the travel time from the storage position p_{i+1} to the retrieval position p_{i+2} (sec.), and t_2 is the pickup/setdown time in the second storage lane of the SR (sec.).

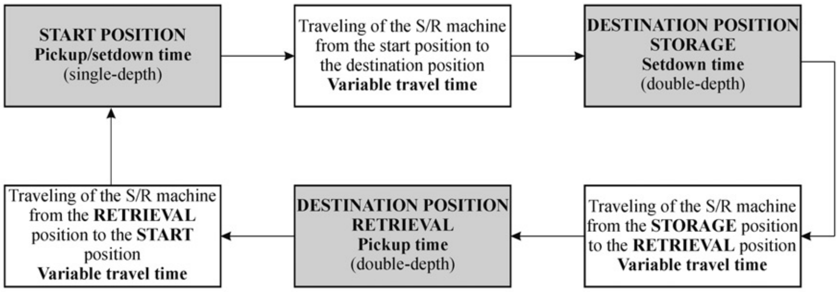

Figure 8. The course of dual command cycle when λ [0, 0.5].

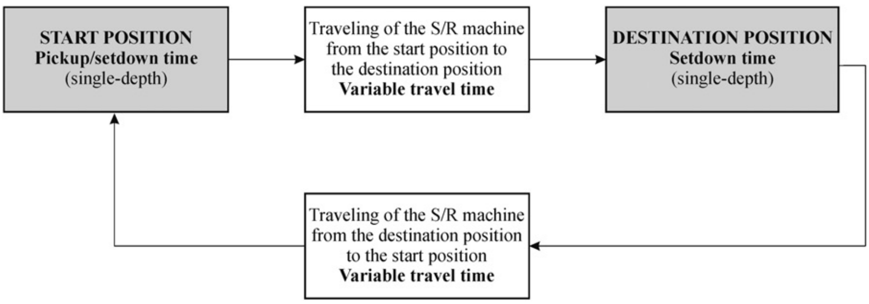

Figure 9. The course of single storage operation when λ [0.5, 1].

5. Cycle time in double-deep AS/RS when α [0.5, 1]

5.1 *Single command cycle (single storage or single retrieval in the cycle)*

5.1.1 *Single storage operation*

For the single storage operation (Figure 9), the single command cycle consists of the time of picking up the TUL at the PD station, travelling to the destination position and setting down the TUL in the first storage lane of the SR, returning empty to the start position and setting down the TUL at the PD station.

The expected single command cycle time E (SC) equals the next expression:

$$E\,(\mathrm{SC}) = 2t_{PD} + 2 \cdot E\big(t_{p_i, p_{i+1}}\big) + t_1, \tag{28}$$

where t_{PD} is the pickup/setdown time at the PD station (sec.), $t_{p_i,\,p_{i+1}}$ is the travel time from the start position p_i to the destination (storage) position p_{i+1} (sec.), and t_1 is the pickup/setdown time in the first storage lane of the SR (sec.).

5.1.2 *Single retrieval operation*

For the single retrieval operation (Figure 10), the single command cycle consists of the time of travelling empty to the destination position in the double-deep SR, retrieving the TUL from the first or the second storage lane of the SR, returning to the start position and setting down the TUL at the PD station. Because the TUL are stored in the first and in the second storage lanes, the access to the second storage lane might be blocked by the

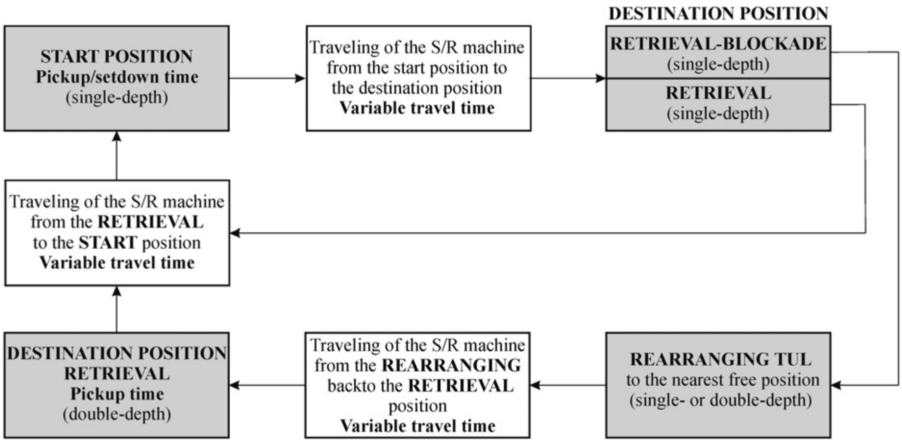

Figure 10. The course of single retrieval operation with rearranging TUL when λ [0.5, 1].

blocking TUL. When a blockade occurs, the S/R machine has to pick up the blocking TUL from the first storage lane and rearrange it to the nearest free position, which can be either in the first or in the second storage lane. Therefore, instead of the regular retrieval time, the additional time for the rearrangement is attached.

The expected single command cycle time E (SC) equals the next expression:

$$E\ (SC) = E\ (SC) + \frac{\alpha}{2} E(R)$$
$$E\ (SC) = 2t_{PD} + 2 \cdot E\big(t_{p_i,p_{i+1}}\big) + (P_1 \cdot t_1 + P_2 \cdot t_2) + \frac{\alpha}{2}[2 \cdot E(t_R) + (P_1 \cdot t_1 + P_2 \cdot t_2)]. \tag{29}$$

t_{PD} Pickup/setdown time at the PD station (sec.).
$t_{p_i,p_{i+1}}$ Travel time from the start position p_i to the destination (retrieval) position p_{i+1} (sec.).
P_1 Probability of retrieval in the first storage lane of the SR.
t_1 Pickup/setdown time in the first storage lane of the SR (sec.).
P_2 Probability of retrieval in the second storage lane of the SR.
t_2 Pickup/setdown time in the second storage lane of the SR (sec.).
t_R Travel time from retrieval position to rearrangement position R (sec.).

The probability P_1, which stands for the condition that the TUL will be pickup/setdown from the first storage lane of the SR, is equal to:

$$P_1 = \frac{P_3}{P_2 + 2P_3} = \frac{2\alpha - 1}{2 - 2\alpha + 2(2\alpha - 1)} = \frac{2\alpha - 1}{2\alpha}. \tag{30}$$

The probability P_2, which stands for the condition that the TUL will be pickup/setdown from the second storage lane of the SR, is equal to:

$$P_2 = \frac{P_2}{P_2 + 2P_3} + \frac{P_3}{P_2 + 2P_3} = \frac{2 - 2\alpha}{2 - 2\alpha + 2(2\alpha - 1)} + \frac{2\alpha - 1}{2 - 2\alpha + 2(2\alpha - 1)} = \frac{1}{2\alpha}. \tag{31}$$

The expected single command cycle time E (SC) now equals:

$$E\,(SC) = 2t_{PD} + 2\cdot E\left(t_{p_i,p_{i+1}}\right) + \left(\left(\frac{2\alpha-1}{2\alpha}\right)\cdot t_1 + \left(\frac{1}{2\alpha}\right)\cdot t_2\right) + \frac{\alpha}{2}\left[2\cdot E(t_R) + \left(\left(\frac{4\alpha-1}{2\alpha}\right)\cdot t_1 + \left(\frac{1}{2\alpha}\right)\cdot t_2\right)\right]. \tag{32}$$

4.2 *Dual command cycle (single storage and single retrieval in the cycle)*

The dual command cycle involves both storage and retrieval operations simultaneously (Figure 11). The dual command cycle time consists of the time to pickup the TUL at the PD station, travel to the destination position and setdown the TUL in the first storage lane of the SR, travel empty to the retrieval position, retrieve the TUL from the first or the second storage lane of the SR, return to the start position and setdown the TUL at the PD station. During the retrieval process rearrangement can occur.

The expected modified dual command cycle time with rearranging TUL equals:

$$E\,(DC)_M = 2t_{PD} + 2\cdot E\big(t_{p_i,p_{i+1}}\big) + t_1 + E\big(t_{p_{i+1},p_{i+2}}\big) + \left(\left(\frac{2\alpha-1}{2\alpha}\right)\cdot t_1 + \left(\frac{1}{2\alpha}\right)\cdot t_2\right) + \frac{\alpha}{2}\left[2\cdot E(t_R) + \left(\left(\frac{4\alpha-1}{2\alpha}\right)\cdot t_1 + \left(\frac{1}{2\alpha}\right)\cdot t_2\right)\right], \tag{33}$$

where $t_{p_{i+1},p_{i+2}}$ is the travel time from the storage position p_{i+1} to the retrieval position p_{i+2} (sec.).

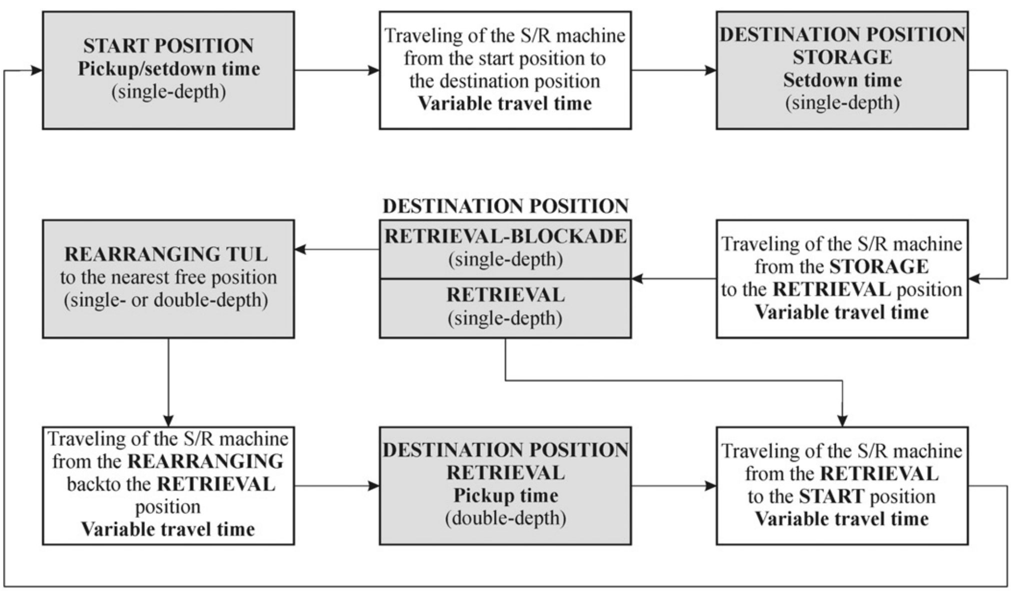

Figure 11. The course of modified dual command cycle when λ [0.5, 1].

6. Simulation model of double-deep AS/RS

To facilitate the evaluation of the performance and comparison of the proposed analytical travel time models for double-deep AS/RS, the discrete event simulation has been employed. The simulation model of double-deep AS/RS consists of a single picking aisle, double-deep SR between picking aisle, S/R machine, I/O location, accumulating conveyors and it has been built using computer software *AutoMod* (Applied Materials AutoMod 2008).

The simulation model begins with the process which marks all storage compartments in the SR according to the prescribed storage area. After creating the list of free storage locations, the first TUL, which is situated in the I/O location and lies in the lower left-hand corner of the picking aisle enter the simulation model. Further on, the TUL receives a sign which belongs to the ith SR location. The S/R machine picks up the TUL from the I/O location, loads it into the hoisted carriage, and transfers it simultaneously in the horizontal and vertical directions to the ith SR location. For the storage operation, the randomised storage policy has been used and the condition that TUL will be stored first in the second SR lane has been applied. In this case there are no rearrangements for storages, and the probability of rearrangement equals to zero. Next, the TUL that has been stored is put on the waiting list by a computer (computer data base), where it waits for the retrieval operation. For the retrieval process the FCFS request selection rules have been used. After the storage operation in the ith SR location, the S/R machine travels to the retrieval location to the jth SR location. During the retrieval process the access to the second storage lane of the SR might be blocked by the blocking TUL in the first storage lane of the SR (a special case of retrieval). In this case the S/R machine rearranges the blocking TUL to the nearest free storage location in the SR, which is selected by the application of Euclidean distance. Next, the S/R machine travels back to the retrieval position, loads TUL into the hoisted carriage and moves it simultaneously in the horizontal and vertical directions to the I/O location, where the TUL departs the system. The dual command cycle is therefore associated with the time for storage and retrieval of TUL and additional time necessary for the rearrangement of the blocked TUL. As a performance measure the average dual command cycle time and consequently the throughput capacity have been used. The throughput capacity represents the number of transactions (stores and retrievals) that the S/R machine can perform in a given time period in the double-deep AS/RS. The throughput capacity is inversely dependent on the average travel cycle time. Therefore the average throughput capacity for the SC and DC in the double-deep AS/RS is given as follows:

$$\begin{aligned} AT &= \frac{r}{2}T(DC)_M + (1-r)T(SC)r = [0,1] \\ \lambda(r) &= \frac{T}{AT}\cdot\eta, \end{aligned} \tag{34}$$

where AT is average travel time [sec.], r is the proportion between SC and DC, T [h] is the analysed time, and η stands for the efficiency of the S/R machine.

7. Double-deep AS/RS (case study)

The double-deep AS/RS is part of the project of designing automated warehouse for a producer of starter batteries. Because of the customer's demand for the high volume utilisation of the SR, the double-deep AS/RS has been proposed. The demanded SR capacity Q was estimated to 2800 pallet places and the throughput capacity λ to 60 single command cycles per hour. The estimation of Q and λ was developed according to the projection of previous production capacity and forecasting capacities in the future. The basic TUL is based on the euro type of palette, with the average height of 1400 mm. Because of the heavy weight of starter batteries, the TUL weighs around 2000 kg. The latter was a very important parameter in designing the AS/RS. Because of the limited layout for the warehouse building (relatively short length of the building), the designers agreed to take advantage of the height of the SR. Therefore, the appropriate construction of the SR with the special requirements for the upright frames and rack beams that are able to hold out the weight of 18 levels in the height has been selected. Also each storage compartment was equipped with the special traverse beam under the supports on palette. The special S/R machine for heavier loads was selected. The S/R machine has special telescopic load attachments that can pickup or setdown the TUL in the single- or double-deep storage lane.

According to the project constraints and customer demands, the following data for the analysis have been applied: $L = 22$ m, $H = 30$ m, $n_x = 20$, $n_y = 18$, $b_w = 1100$ mm, $b_d = 1200$ mm, $a_w = 1500$ mm, the I/O location $(x = -1000, y = 1000)$ mm, $v_x = 1.5$ m/s, $v_y = 1$ m/s, $a_x^+ = 0.5$ m/s^2, $a_y^+ = 0.5$ m/s^2, $a_x^- = 0.5$ m/s^2, $a_y^- = 0.5$ m/s^2, $v_t = 0.7$ m/s, $t_{PD} = 3.43$ sec., $t_1 = 3.43$ sec. and $t_2 = 6.86$ sec.

7.1 *Analyses and evaluation of results*

The modified dual command cycle times and throughput capacities for the selected double-deep AS/RS, shown in Tables 1–4, are given on the basis of the performed

Table 1. The modified dual command cycle time analysis for the proposed double-deep AS/RS.

	Modified dual command cycle time (sec.)			Throughput performance of DC_M (TUL/hour)		
α	Analytical $E(DC)_M$	Percent error	Simulation $T(DC)_M$	Analytical $\lambda(DC)_M$	Percent error	Simulation $\lambda(DC)_M$
0.55	51.69* **71.27****	1.16***	50.87 **70.45**	**91**	−1.09	**92**
0.60	51.86 **71.36**	0.79	51.31 **70.80**	**91**	−1.09	**92**
0.65	52.06 **71.5**	0.31	51.84 **71.28**	**91**	0.00	**91**
0.70	52.26 **71.69**	−0.36	52.53 **71.95**	**90**	0.00	**90**
0.75	52.50 **71.94**	−0.92	53.18 **72.61**	**90**	1.12	**89**
0.80	52.79 **72.26**	−1.42	53.84 **73.3**	**90**	2.27	**88**
0.85	53.14 **72.66**	−1.96	54.6 **74.11**	**89**	2.30	**87**
0.90	53.64 **73.21**	−2.27	55.34 **74.91**	**89**	2.30	**87**
0.95	54.52 **74.16**	−2.82	56.61 **76.31**	**87**	2.35	**85**

Note: *The variable share of cycle time. **The cycle time associated with load handling. ***Percent error between continuous and discrete model versus the fill-grade factor α for the double-deep AS/RS. Factor r was set to 1 and the efficiency of the S/R machine was set to 0.90.

Table 2. Cycle time and capacity analyses of the proposed double-deep AS/RS with $\alpha = 0.85$.

	Cycle time (sec.)				Throughput performance (TUL/hour)		
	Analytical	Percent error	Simulation		Analytical	Percent error	Simulation
E(SC)	**47.19**	(1.51)*	**46.49**	λ(SC)	**69**	(–1.43)	**70**
$E(\mathrm{DC})_M$	**72.66**	(–1.96)	**74.11**	$\lambda(\mathrm{DC})_M$	**89**	(2.30)	**87**
r		AT		r		$\lambda(r)$	
0.0	**47.19**	(1.51)	**46.49**	0.0	**69**	(–1.43)	**70**
0.1	**46.10**	(1.22)	**45.55**	0.1	**70**	(–1.41)	**71**
0.2	**45.02**	(0.93)	**44.60**	0.2	**72**	(–1.37)	**73**
0.3	**43.93**	(0.62)	**43.66**	0.3	**74**	(0.00)	**74**
0.4	**42.85**	(0.30)	**42.72**	0.4	**76**	(0.00)	**76**
0.5	**41.76**	(–0.03)	**41.77**	0.5	**78**	(0.00)	**78**
0.6	**40.67**	(–0.38)	**40.83**	0.6	**80**	(1.27)	**79**
0.7	**39.59**	(–0.75)	**39.89**	0.7	**82**	(1.23)	**81**
0.8	**38.50**	(–1.13)	**38.94**	0.8	**84**	(1.20)	**83**
0.9	**37.42**	(–1.53)	**38.00**	0.9	**87**	(2.35)	**85**
1.0	**36.33**	(–1.96)	**37.06**	1.0	**89**	(2.30)	**87**

*Percent error between continuous and discrete model versus the factor r for the double-deep AS/RS with the fill-grade factor $\alpha = 0.85$.

Table 3. Cycle time and capacity analyses of the proposed double-deep AS/RS with $\alpha = 0.90$.

	Cycle time (sec.)				Throughput performance (TUL/hour)		
	Analytical	Percent error	Simulation		Analytical	Percent error	Simulation
E(SC)	**47.19**	(1.51)*	**46.49**	λ(SC)	**69**	(–1.43)	**70**
$E(\mathrm{DC})_M$	**73.21**	(–2.27)	**74.91**	$\lambda(\mathrm{DC})_M$	**89**	(2.30)	**87**
r		AT		r		$\lambda(r)$	
0.0	**47.19**	(1.51)	**46.49**	0.0	**69**	(–1.43)	**70**
0.1	**46.13**	(1.20)	**45.59**	0.1	**70**	(–1.41)	**71**
0.2	**45.07**	(0.87)	**44.68**	0.2	**72**	(–1.37)	**73**
0.3	**44.01**	(0.54)	**43.78**	0.3	**74**	(0.00)	**74**
0.4	**42.96**	(0.19)	**42.88**	0.4	**75**	(–1.32)	**76**
0.5	**41.90**	(–0.18)	**41.97**	0.5	**77**	(0.00)	**77**
0.6	**40.84**	(–0.56)	**41.07**	0.6	**79**	(0.00)	**79**
0.7	**39.78**	(–0.96)	**40.17**	0.7	**81**	(0.00)	**81**
0.8	**38.72**	(–1.38)	**39.26**	0.8	**84**	(1.20)	**83**
0.9	**37.66**	(–1.81)	**38.36**	0.9	**86**	(2.38)	**84**
1.0	**36.61**	(–2.27)	**37.46**	1.0	**89**	(2.30)	**87**

*Percent error between continuous and discrete model versus the factor r for the double-deep AS/RS with the fill-grade factor $\alpha = 0.90$.

Table 4. Cycle time and capacity analyses of the proposed double-deep AS/RS with $\alpha = 0.95$.

	Cycle time (sec.)				Throughput performance (TUL/hour)		
	Analytical	Percent error	Simulation		Analytical	Percent error	Simulation
E(SC)	**47.19**	(1.51)*	**46.49**	λ(SC)	**69**	(–1.43)	**70**
E(DC)M	**74.16**	(–2.82)	**76.31**	λ(DC)M	**87**	(2.35)	**85**
r		AT		r		λ(r)	
0.0	**47.19**	(1.51)	**46.49**	0.0	**69**	(–1.43)	**70**
0.1	**46.18**	(1.14)	**45.66**	0.1	**70**	(–1.41)	**71**
0.2	**45.17**	(0.77)	**44.82**	0.2	**72**	(0.00)	**72**
0.3	**44.16**	(0.38)	**43.99**	0.3	**73**	(–1.35)	**74**
0.4	**43.15**	(–0.02)	**43.16**	0.4	**75**	(0.00)	**75**
0.5	**42.14**	(–0.44)	**42.32**	0.5	**77**	(0.00)	**77**
0.6	**41.12**	(–0.88)	**41.49**	0.6	**79**	(1.28)	**78**
0.7	**40.11**	(–1.33)	**40.66**	0.7	**81**	(1.25)	**80**
0.8	**39.10**	(–1.81)	**39.82**	0.8	**83**	(2.47)	**81**
0.9	**38.09**	(–2.30)	**38.99**	0.9	**85**	(2.41)	**83**
1.0	**37.08**	(–2.82)	**38.16**	1.0	**87**	(2.35)	**85**

*Percent error between continuous and discrete model versus the factor r for the double-deep AS/RS with the fill-grade factor $\alpha = 0.95$.

analyses. Analyses have been conducted for the chosen AS/RS mentioned before, under the condition of possible rearrangement of TUL during the retrieval process. In order to receive the best representative average travel time, the simulation results correspond to a large number of runs for every single variant ($\alpha = 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.95$) of the double-deep AS/RS. According to the simulation results presented in Tables 1–4, the performance comparison between the proposed analytical models has been analysed.

According to the results in Table 1, the proposed analytical travel time models for the double-deep AS/RS, demonstrate good performances with regard to the results of simulation analysis. According to the increased fill-grade factor α, an increasing trend of the modified dual command cycle time can be noticed for both proposed models and the simulation model. This relationship can be explained by the increasing length of the rearrangement distances, which causes the enlargement of the rearrangement time and consequently the cycle time. Because the throughput performance is inversely dependant on the cycle time, the decreasing trend of throughput capacities with regard to the increased fill-grade factor α, can be noticed.

In general, according to the difference between the proposed analytical travel time models and the simulation model, small deviations in the range of –2.82% are noticed. The latter show that the proposed analytical travel time models demonstrate satisfactory deviations of modified dual command cycle time and throughput performances for the selected double-deep AS/RS.

Because AS/RS usually have the fill-grade factor $\alpha = 0.85$, 0.90 and 0.95 the analyses in Tables 2–4 correspond to the selected double-deep AS/RS with the chosen α.

According to the demanded SR capacity Q and throughput capacity λ, the calculated single command cycle times (single storage operation) and modified dual command cycle times (single storage and single retrieval operation) and the throughput performance of the selected double-deep AS/RS correspond to the desired 60 single command cycles per hour. The throughput performance of the single command cycle in the case of proposed analytical travel time models is $\lambda(SC) = 69$ TUL/hour, whereas is the throughput capacity of simulation model $\lambda(SC) = 70$ TUL/hour. The deviation in the throughput performance between the proposed analytical model and simulation model, for the selected double-deep AS/RS ($\alpha = 0.85, 0.90$ and 0.95), is in the range of –1.43%. According to the results of the modified dual command cycle and the throughput performance, the deviation between the proposed analytical model and simulation model becomes higher and is in the range of 2.35% ($\alpha = 0.95$), which is still satisfactory. It must be emphasised that the demanded throughput performance λ also can be increased by utilising both the single and dual command cycles, which is expressed in Tables 2–4 with the factor r. For example if the factor $r = 0.6$ ($\alpha = 0.95$), the throughput performance is increased from 69 (70) to 79 (78) TUL/hour. The maximal throughput performance is expressed with the factor $r = 1$, which means that only dual command cycles are utilised.

Due to the demanded rack capacity, two double-deep AS/RS were chosen, which give us the desired rack capacity $Q = 2880$ pallet places.

8. Conclusions

In this paper, the proposed analytical travel time models for the unit-load double-deep AS/RS are presented. In contrast to other researchers who have been occupied in their analytical travel time models with uniform velocities only, the real operating characteristics of the S/R machine and FEM 9.851 guideline have been used in the proposed analytical travel time models. In the proposed models it has been assumed that according to the single shuttle system (only one TUL could be placed on the hoisted carriage), the rearrangement during the storage process does not occur. On the contrary, in the case of retrieval process the access to the second SR lane might be blocked by the blocking TUL in the first storage lane. The problem can be solved by rearranging the blocking TUL to the nearest free storage location in the SR. Thus, considering the randomised storage assignment rule and the above mentioned conditions, the proposed analytical travel time models have been developed.

The cycle time and the performance capacity of the selected double-deep AS/RS have been examined in order to investigate the efficiency of the proposed models in comparison with the simulation model of double-deep AS/RS. In general, according to the difference between the proposed analytical travel time models and the simulation model, small deviations in the range of – 2.82% are noticed. Therefore the proposed analytical travel time models demonstrate good performances for designing the double-deep AS/RS and could be a very useful tool for the professionals in practice.

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