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ORIGINAL ARTICLE

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Computational approach to contact fatigue damage initiation analysis of gear teeth flanks

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Abstract The paper describes a general computational model for the simulation of contact fatigue-damage initiation in the contact area of meshing gears. The model considers the continuum mechanics approach, where the use of homogenous and elastic material is assumed. The stress field in the contact area and the relationship between the cyclic contact loading conditions and observed contact points on the tooth flank are simulated with moving Hertzian contact pressure in the framework of the finite element method analysis. An equivalent model of Hertzian contact between two cylinders is used for evaluating contact conditions at the major point of contact of meshing gears. For the purpose of fatigue-damage analysis, the model, which is used for prediction of the number of loading cycles required for initial fatigue damage to appear, is based on the Coffin-Manson relationship between deformations and loading cycles. On the basis of computational results, and with consideration of some particular geometrical and material parameters, the initiation life of contacting spur gears in regard to contact fatigue damage can be estimated.

Keywords Contact fatigue · Crack initiation · Numerical modelling · Gear teeth flanks

1 Introduction

The fatigue process of mechanical elements is a material characteristic and depends upon cyclic plasticity, local deformation, dislocation motion and formation of micro- and macro-cracks and their propagation. Contact fatigue is extremely important for all engineering applications involving localized contacts such as gears, brakes, clutches, rolling bearings, wheels, rails, screws and riveted joints. The repeated rolling and/or sliding contact conditions cause permanent damage to the material due to accumulation of deformation. Contact fatigue process can be divided into two main parts: (1) initiation of micro-cracks due to local accumulation of dislocations, high stresses in local points, plastic deformation around inhomogeneous inclusions or other imperfections on or under contact surface; (2) crack propagation, which causes permanent damage to a mechanical element, i.e. exceeding the fracture toughness of the material. The main aim of this paper is to model contact fatigue crack initiation. Although modelling of contact fatigue initiation is often supported by experimental investigations, most of the work published on contact fatigue is theoretical [1, 2, 5, 12]. The work of Mura and Nakasone [21] and Cheng et al. [5] represent a large step forward in the field of developing physical and mathematical models concerning fatigue-damage initiation. Practical applicability of those models in engineering is, however, limited. Large numbers of different materials and geometric parameters are needed to determin the calculation number of loading cycles for fatigue-damage initiation [5]. Additionally, those parameters differ for different materials, geometries and loading spectra [5]. The applicability of analytical methods is limited to idealized engineering problems and they are based upon well known theoretical methods for determining fatigue-damage initiation strain-life methods including Coffin-Manson's hypothesis, Morrow's analysis, Smith-Watson-Topper (SWT) method, etc. [2, 4, 9, 11, 23, 24].

The significance of dealing with different mechanical elements and constructions and different approaches to sizing and endurance control proves that those problems are of great interest nowadays [2, 3, 7, 9, 10, 23]. For designing machines and devices, dimensioning with respect to service life is increasingly taken into account. This also applies to gearing which is still today one of the very important components of almost all machines. Two kinds of teeth damage can occur on gears under repeated loading due to fatigue: namely the pitting of gear teeth

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flanks and tooth breakage in the tooth root. Various studies have analysed numerical approaches to bending fatigue life in tooth root [2, 13], as well as pitting resistance of gear teeth flanks [10, 12, 14–17, 22]. The effect of different surface treatments on the fatigue behaviour at the tooth root of spur gears has also been investigated [3].

In our previous research work, the main focus was on the bending and pitting analysis of spur gears [2, 10, 12–17] in regards to fatigue crack growth. The initiation phase of the material fatigue process is now included. This paper describes a computational model for contact fatigue crack initiation in the contact area of gear teeth flanks. In the presented paper, the focus is on the numerical determination of damage initiation at gear teeth flanks. The most common methods of gear design are based on conventional standard procedures like DIN [6], AGMA [1] and ISO. Although the standards for calculation are the most up-to-date methods available, they do not give detailed information of the fatigue life of gears. However, all standard models are, in some manner, rough and do not give enough accurate results because they do not take into account the actual conditions. Therefore, the development of models and procedures for calculations of fatigue-damage initiation of spur gears seems to be a good choice. For this purpose, a general computational model of contact fatigue initiation has been developed [24]. Computational models to fatigue initiation analyses are based upon the method which determines the ratio between the specific deformation and the number of loading cycles, often referred to as local stress-strain method [11, 18, 25]. Lately, a connection between the results, obtained by means of the finite element method (FEM) and/or the boundary element method (BEM), and those, obtained by the analyses of material fatigue [4, 9, 24], can be traced. Due to the complexity of contact fatigue initiation at gear teeth flanks, the generalized model of contact fatigue initiation, presented in [24] is appropriately modified for this particular application. The loading cycle during meshing of a gear pair needs to be determined first. The Hertzian boundary conditions are evaluated at characteristic points along the engagement line by means of an equivalent model of two cylinders. The changing characteristics of gear teeth sliding along the engagement line have been considered by means of a corresponding coefficient of friction. The resulting stress loading cycle of meshing gears is then used for the contact fatigue analysis. The influence of the type and quality of the contact surface treatment has also been considered—in computational simulation this is usually done by means of a corresponding fatigue limit parameter.

The process of surface pitting can be visualized as the formation of small surface initial cracks which grow under repeated contact loading. Eventually, the crack becomes large enough for unstable growth to occur, which causes the material surface layer to break away. The resulting void is called the surface pit (Fig. 1).

The number of stress cycles, N, required for the pitting of a gear teeth flank to occur can be determined from the number of stress cycles, N_i, required for the appearance of the initial crack in the material and the number of stress cycles, N_p, required for a crack to propagate from the initial to the critical crack length, when the final failure can be expected to occur:

$$N = N_i + N_p. \tag{1}$$

However, the prime aim of the present study is to present a computational model for prediction of contact fatigue initiation—e.g. N_i in Eq. (1)—which is based on continuum mechanics, cyclic contact loading and characteristic material fatigue parameters. The material model is assumed as being homogeneous, without the imperfections such as inclusions, asperities, roughness, residual stresses, etc., which often occur in mechanical elements. Moving contact load is used for simulation of the cyclic loading in fatigue crack initiation analysis of the meshing of gears.

2 Description of the problem and computational model

2.1 Contact fatigue process at meshing gears

Initial surface cracks leading to the pitting of gears are generally observed to appear in the contact areas where high normal contact pressure is combined with significant sliding velocities, which result in additional frictional loading of the surface material layer. There are several locations where pitting is apt to occur. However, the most critical contact loading conditions for initial crack formation and propagation are identified as the rolling contact with sliding, where the contact sliding, and with that the effect of friction, is opposite to the direction of the rolling contact motion [8, 17]. Thus, the worst contact loading conditions appear when the gear teeth are in contact at the inner point of single teeth pair engagement (point B, Fig. 4), where the surface-breaking initial cracks are expected to develop first. Of course, this fact is valid for the pinion gear in the case of revolutions reduction (reduction gear). Furthermore, pinions are apt to pit rather than gears

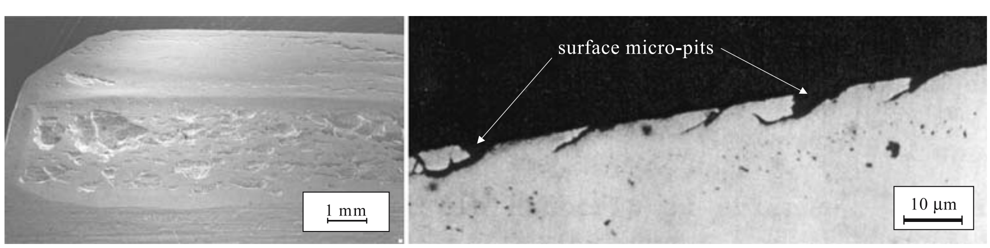

Fig. 1 Typical surface micro-pits on gear tooth flank [17]

for two reasons [8]. First, the pinion is ordinarily the *driver* (reduction gear). The directions of sliding are such that sliding is away from the pitch line on the driver and toward the pitch line on the driven member (Fig. 2). So, the sliding motion on the driver tends to pull metal away from the pitch line. On the other hand, on the gear the sliding tends to compress the metal at the pitch line. Finally, the initial cracks that form when a surface is severely loaded have a tendency to intersect at the pitch line on the driver, while on the driven member they do not. Secondly, the pinion, being smaller, has obviously more cycles of operation than the gear. The slope of the fatigue curve makes the part with the most cycles the most apt to fail [8].

Basic parameters, influencing contact fatigue life of spur gears can be summarized as hertzianpressure on the tooth flank and sliding conditions alongside the engagement line.

Pressure on the tooth flank can be estimated with classical Hertzian theory using an equivalent contact model (Fig. 3), and can be expressed analytically by [6],

$$\sigma_H = \sqrt{\frac{E}{2 \cdot b \cdot \pi \cdot (1-\nu^2)} \cdot \frac{F_N}{R^*}}, \tag{2}$$

where σ_H is Hertzian pressure, E is Young's elastic modulus, b is the gear width, ν is Poisson's ratio, F_N is the normal force on the tooth flank and R^* is the equivalent radius curvature (Fig. 3).

Sliding alongside engagement line is due to a difference between tangential components of velocity in the particular gear contact points (Fig. 2a). As a rule, the maximum value of tangential velocity difference occurs when the root of the teeth (dedendum) and the top of the counter-teeth (addendum) are meshing. Wear is usually given as a numerical value (parameter) and depends on the specific sliding coefficient ψ (Fig. 2b).

However, additional, and also very important parameters are: temperature on the tooth flank, thickness of the oil film in the EHD lubrication regime, residual stresses in the area near the top of the gear tooth flank surface, local geometry of the tooth flank, roughness of the contacting profile etc.

Nevertheless, one of the main difficulties in numerical modelling of contact fatigue at spur gears seems to be the detailed determination of operational loading and/or strain–stress cycles of the meshing gears. Since the loading cycles are very important for the determination of fatigue life and directly influence the computational procedure of the initiation damage definition, they must be precisely determined. For this purpose, the following equivalent numerical contact model is introduced.

2.2 Equivalent numerical model

For computational determination of fatigue crack initiation at gear teeth flanks, an equivalent contact model (Hertzian theory) [10, 15, 17, 19, 24] is used (Fig. 3). The equivalent cylinders have the same radii, as the curvature radii of gear flanks at the observed point (the inner point of single teeth pair engagement—point B).

According to Hertzian theory [19], the distribution of normal contact pressure in the contact area can be determined by

$$p(x) = \frac{2F_N}{\pi a}\sqrt{1-\frac{x^2}{a^2}}, \tag{3}$$

where F_N is the normal contact force per unit width, a is half length of the contact area, which can be determined from [19]

$$a = \sqrt{\frac{4F_N R^*}{\pi E^*}}, \tag{4}$$

where E^* and R^* are the equivalent Young's elastic modulus and the equivalent radius, respectively, defined as [19]

$$\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}, \tag{5}$$

$$\frac{1}{R^*} = \frac{1}{R_1} + \frac{1}{R_2}, \tag{6}$$

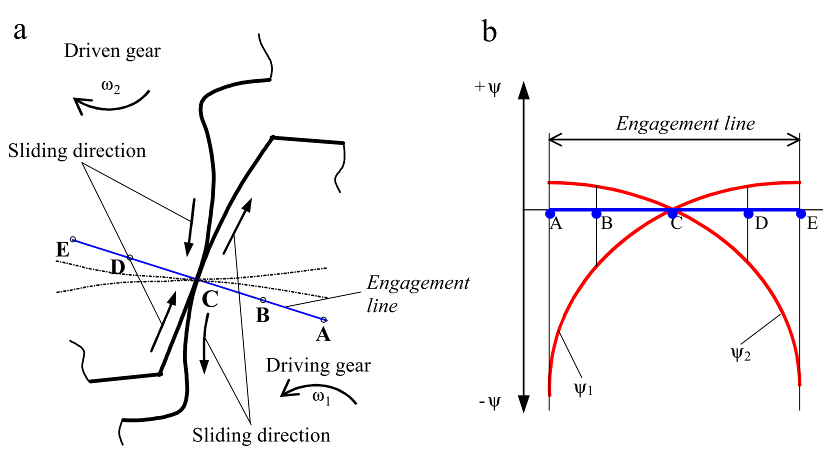

Fig. 2 Specific sliding along engagement line: **a** meshing conditions at tooth flank, **b** diagram of specific sliding along engagement line

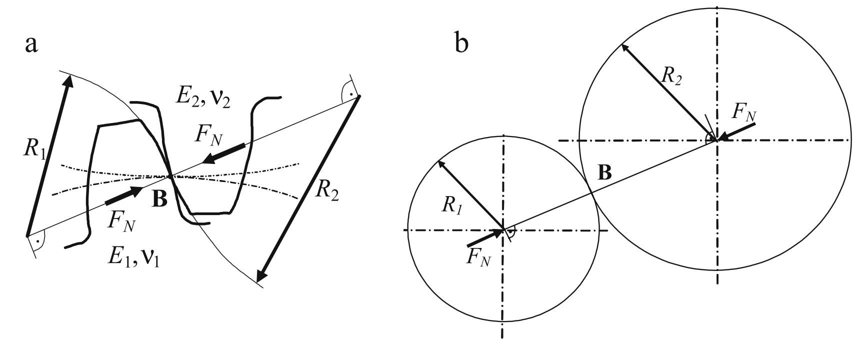

Fig. 3 Model transformation of meshing gears (**a**) to the equivalent model of two cylinders for the meshing point B (**b**)

where E_1, R_1, ν_1 and E_2, R_2, ν_2 are the Young modulus, the curvature radii and Poisson ratio of the contacting cylinders (see Fig. 2). The maximum contact pressure p_0=p(x=0) can then be determined as [19]

$$p_0 = \sqrt{\frac{F_N E^*}{\pi R^*}}. \tag{7}$$

In analysing real mechanical components, some partial sliding occurs during time dependent contact loading, which can originate from different effects (complex loading conditions, geometry, surface etc.) and it is often modelled by traction force due to the pure Coulomb friction law [19]. In the analysed case, frictional contact loading $q(x)$ is a result of the traction force action (tangential loads) due to the relative sliding of the contact bodies and is determined here by utilizing the previously mentioned Coulomb friction law [19]

$$q(x) = \mu \cdot p(x), \tag{8}$$

where μ is the coefficient of friction between contacting bodies.

In regards to a general case of elastic contact between two deformable bodies in a standstill situation, the analytical solutions are well known. However, using general Hertzian equations [19], it is difficult to provide the loading cycle history and/or simulation of a contact pressure distribution of moving contact in the analytical manner. Therefore, the finite element method (FEM) is used for evaluating two-dimensional friction contact with the aim of prescribing loading cycles at the gears.

2.3 Fatigue crack initiation analysis

For accurate determination of the service life of gears, loadings, which are in most cases random loadings of variable amplitude, and the geometry of the gears and properties of materials, of which the gears are made and which are not known to be constants, have to be taken into account. The more precise the modelling of these input parameters, the more precise and reliable are the results.

In practice, the following two basic problems arise when calculating the service life. Firstly, during designing, the individual elements and the entire products are optimized particularly with respect to the service life. The basic requirement is that the service lives of the individual elements are approximately equalized. In this case, the model of occurrence of defects (crack initiation) and the model of crack propagation are important [2]. Secondly, a defect, i.e. an initial crack, is detected during periodic inspection by a non-destructive method. If the component concerned is not on stock or a considerable period of time would be necessary to manufacture it, the interesting fact is to know how long the damaged component will operate with full rated loading and/or what the loading for the desired service life is, i.e. the time necessary for the manufacture of a new component – gear.

When the stress loading cycles are determined, the fatigue analysis for each observed material point can be performed. The methods for fatigue analysis are most frequently based on the relation between deformations, stresses and the number of loading cycles and are usually modified to fit the nature of the stress cycle, e.g. repeated or reversed stress cycle (Zahavi and Torbilo [25]). The number of stress cycles required for a fatigue crack to appear, can be determined iteratively with the strain-life method ε-N, where the relationship between the specific deformation increment, $\Delta\varepsilon$, and the number of loading cycles, N_f, is fully characterized with the following equation [25]

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_a}{E} + \frac{\Delta\varepsilon_p}{2} = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c, \tag{9}$$

where σ'_f is the fatigue strength coefficient, b the strength exponent, ε'_f the fatigue ductility coefficient and c is the fatigue ductility exponent. Generally, the following modified approaches of the strain-life method are most often used for fatigue calculations: Coffin-Manson's hypothesis (ε-N method), Morrow's analysis, Smith-Watson-Topper (SWT) method [25]. According to Morrow, the relationship between strain amplitude, ε_a, and pertinent number of load cycles to failure, N_f, can be written as [21]

Table 1 Data set for the gear pair

Parameter	Pinion	Pinion and Gear	Gear
Normal module		m_n=4,5 mm	
Number of teeth	z_1=16		z_2=24
Pressure angle on pitch line		α_n=20°	
Helical angle		β_0=0°	
Coefficient of profile displacement	x_1=0,182		x_2=0,171
Centre distance		a=91,5 mm	
Tooth width		b_1=b_2=14 mm	
Pitch diameter	d_1=72 mm		d_2=108 mm
Material of gear pair		42CrMo4, case hardened	

$$\varepsilon_a = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c, \tag{10}$$

where E is the elastic modulus, σ'_f is the fatigue strength coefficient, ε'_f is the fatigue ductility coefficient, b is exponent of strength and c is the fatigue ductility exponent, respectively.

Morrow's equation with mean stress, σ_m, correction [25] is expressed as:

$$\varepsilon_a = \frac{\left(\sigma'_f = \sigma_m\right)}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c, \tag{11}$$

According to Coffin-Manson, this relationship can be simplified as

$$\varepsilon_a = 1,75\frac{\sigma_{UTS}}{E}N_f^{-0,12} + 0.5D^{0,6}N_f^{-0,6}, \tag{12}$$

where σ_{UTS} is the ultimate tensile stress and D is the ductility, defined as

$$D = \ln\frac{A_0}{A_{fracture}} \cong \varepsilon_{fracture}.$$

The Smith-Watson-Topper method considers the influence of the mean stress value and can be defined as [21]

$$\sigma_1^{\max}\frac{\Delta\varepsilon_1}{2} = \sigma'_f\varepsilon'_f\left(2N_f\right)^{b+c} + \frac{\sigma'^2_f}{E}\left(2N_f\right)^{2b}, \tag{13}$$

where $\Delta\varepsilon_1$ is the amplitude of the maximum principal deformation and $\sigma_1^{\max}$ is the maximum value of principal stresses in the direction of maximum principal deformation.

All the described fatigue life models used in this work are actually based on the strain-life method. Once a local stress or strain-time history is established, a fatigue analysis method has to be applied. Furthermore, material properties are introduced as material fatigue data from tests [20]. The strain life approach has an advantage over stress versus the number of loading cycles (S-N) method, even in high cycle application, due to its less scatter-prone materials data. Assumptions that have been made in these fatigue life models include the following: the models are uni-axial,

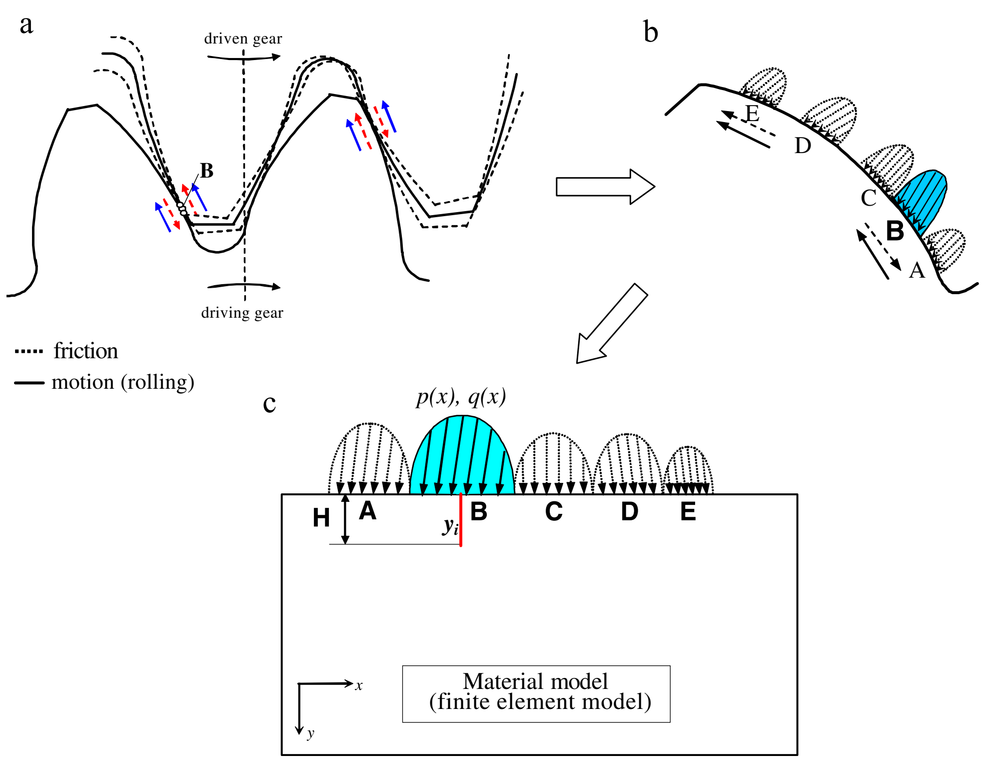

Fig. 4 Moving contact loading configurations in respect to the initial crack position: **a** meshing of gear pair, **b** equivalent contact model of meshing gears and **c** material model used for determination of loading cycles

Table 2 Characteristic magnitude at meshing points using for determination of the contact loading cycle upon the meshing of two gears (DIN 3990)

Hertzian pressure	Equivalent radius	Contact width	Friction coefficient
p_0^A=1,402 MPa	R_A^*=3,8999 mm	$2a^A$=0.1843 mm	μ_A=0.1088
p_0^{AB}=1,148 MPa	R_{AB}^*=5,8198 mm	$2a^{AB}$=0.2251 mm	μ_{AB}=0.0975
p_0^B=1,453 MPa	R_B^*=7,2562 mm	$2a^B$=0.3555 mm	μ_B=0.1075
p_0^C=1352 MPa	R_C^*=8,3821 mm	$2a^C$=0.3821 mm	μ_C=0.0000
p_0^D=1325 MPa	R_D^*=8,7289 mm	$2a^D$=0.3900 mm	μ_D=0.1004
p_0^E=1,000 MPa	R_E^*=7,6637 mm	$2a^E$=0.2584 mm	μ_E=0.0868

which means that just one principal stress occurs in one direction (single stress vector). However, for more complex (multiaxial) loading conditions, multiaxial critical plane analysis should be applied.

Before damage can be determined and summed for each cycle, a certain amount of correction needs to take place; the main correction being the conversion of purely elastic stresses and strains to elasto-plastic stresses and strains. In this work, plasticity is accounted for in the crack initiation method by the Neuber method [11, 25]. The elastic stresses and strains are looked up on the elastic line and then corrected to fall onto the cyclic stress strain curve to determine the elastic-plastic stresses and strains. It is then this elastic-plastic strain that is used to determine damage on the strain-life damage curve. Neuber's elastic-plastic correction is based on a simple principle that the product of the elastic stress and strain should be equal to the product of the elastic-plastic stress and strain from the cyclic stress-strain curve [11, 25]:

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma}{2E} + \left[\frac{\Delta\sigma}{2K'}\right]^{1/n'} \Delta\sigma \cdot \Delta\varepsilon = E \cdot \Delta\varepsilon_e^2, \tag{14}$$

where $\Delta\varepsilon$, $\Delta\sigma$ are incremental values of strain and stress, E is the elastic modulus, $\Delta\varepsilon_e$ is the incremental value of elastic strain, n' is the material hardening exponent and K' is the material strength coefficient. Using an iterative method, the elastic-plastic stress and strain can be determined.

3 Practical application of the computational model

The proposed computational model is applied to the analysis of the fatigue crack initiation on the tooth flank of a real gear pair with the material and geometric data set given in Table 1.

If determination of the loading cycle in the tooth root fatigue process seems to be a simple procedure (model of cantilever beam subjected to pulsating force), the determination of loading cycles at the contact region of meshing gears (at teeth flanks) appears to be a very difficult task. Indeed, the worst case of pitting phenomena, i.e. the area around the inner point of a single teeth pair engagement, is often simulated with the maximum value of contact pressure via point force applied to this point (B). However, this is not in agreement with the fact that the loading cycles are sufficient for evaluating fatigue process. For the treated gear pair, contact point B can be identified with the equivalent radius, as well as other characteristic points (see Fig. 4 and Table 2). In the following, the prediction of time-dependent loading cycles and the evaluation of the number of critical loading cycles of real meshing gears are presented.

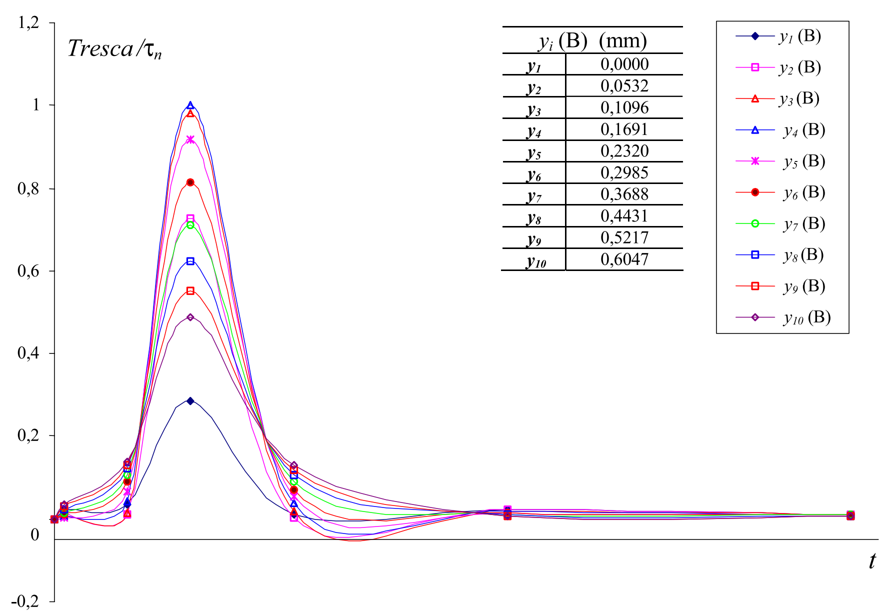

Fig. 5 Normalised Tresca comparative stress cycles on the meshing gears, at and under contact point B

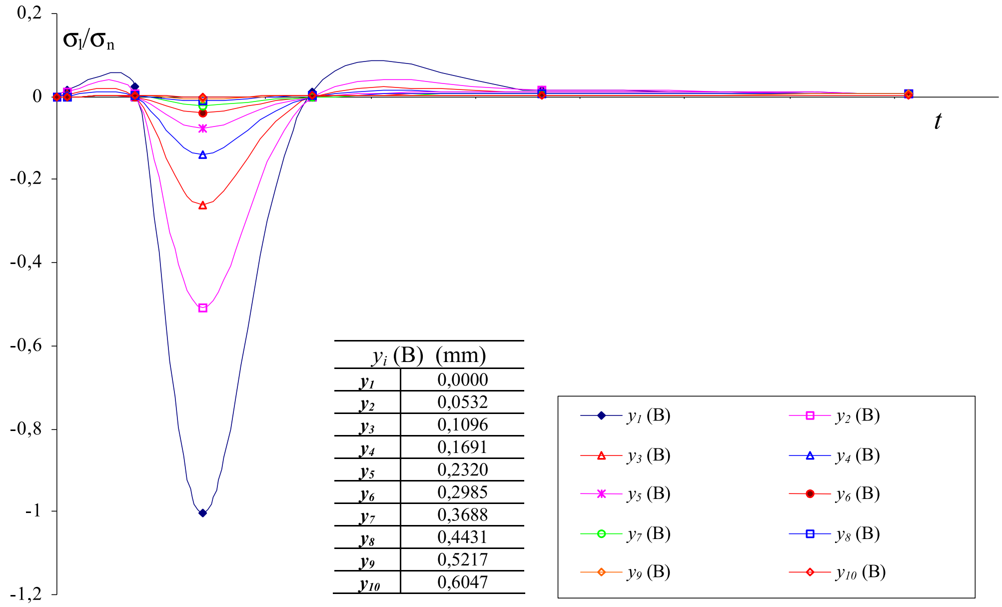

Fig. 6 Normalized maximal principal stress σ_1 cycles at the meshing gears, on and under contact point B

3.1 Moving contact loading of meshing gears

A comprehensive model for contact fatigue life prediction of single mechanical elements should consider the time history of applied contact loads with regard to both their magnitude and shape. For a more realistic simulation of the fatigue crack initiation at the teeth flank, it is necessary to consider the influence of moving contact in the vicinity of the expected initial crack(s). The moving contact can be simulated with different loading configurations, as shown in Fig. 4. FE analysis is used only for contact stress calculation and loading cycle determination.

Six characteristic contact points (A, A–B, B, C, D and E; Fig. 4) of the meshing gears have been considered, each with the specific normal $p(x)$ and tangential $q(x)$ contact loading distributions. The values of the basic contact parameters presented in Table 2 have been considered using Hertzian theory and the DIN 3990 [6] standard procedure.

If the values of the equivalent roller radius correspond to the curvature radii of the teeth flanks, then with proper estimation of the rotation velocity of rollers, one can determine the conditions at each characteristic meshing point. Those magnitudes are then used for determination of contact loading cycles for meshing gears (particularly for the observed material point B; Fig. 4). An idea which is put forward here, is that the loading cycle can be defined as the corresponding value of stresses at each of the observed meshing points (A, A–B, C, B, D, E), regarding critical point B (see Fig. 4). In that way, a time dependent loading cycle, focused on point B, can be achieved. Of course, loading cycles for other characteristic points can be defined as well, but the main focus in this work is on the inner point of a single teeth-pair engagement.

To obtain the distributions of loading cycles presented for each observed contacting point and under it, the stress field was defined at the crossing of the contact loading (actually, at the meshing of the gears or at the one revolution of gear pair). The gear tooth surface is not continuously being utilized; each part of the tooth surface is utilized for only short period of time. Generally, the obtained times are very short, since in the presented case, the rotational velocity of the pinion is 2,175 rpm. In the case of Tresca stress comparative cycles, the time dependent normalized distribution of loading cycles, considering surface and under surface layers (Fig. 4), are presented in Fig. 5. Considering the maximal principal stresses σ_1, the distributions of the loading cycles are presented in Fig. 6.

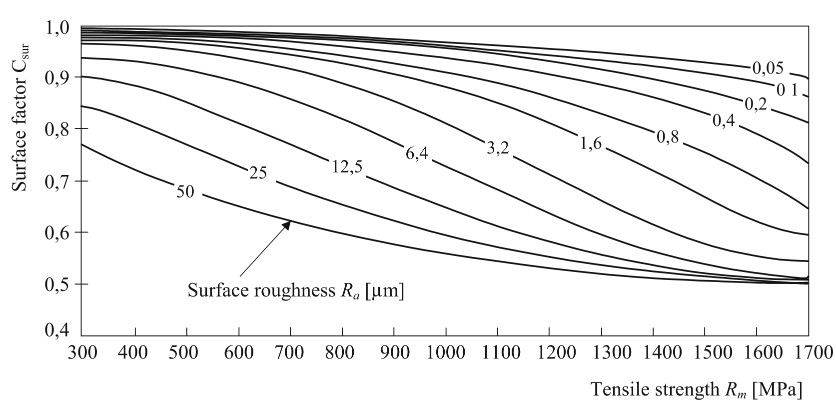

Fig. 7 Surface finish correction factor C_{sur}

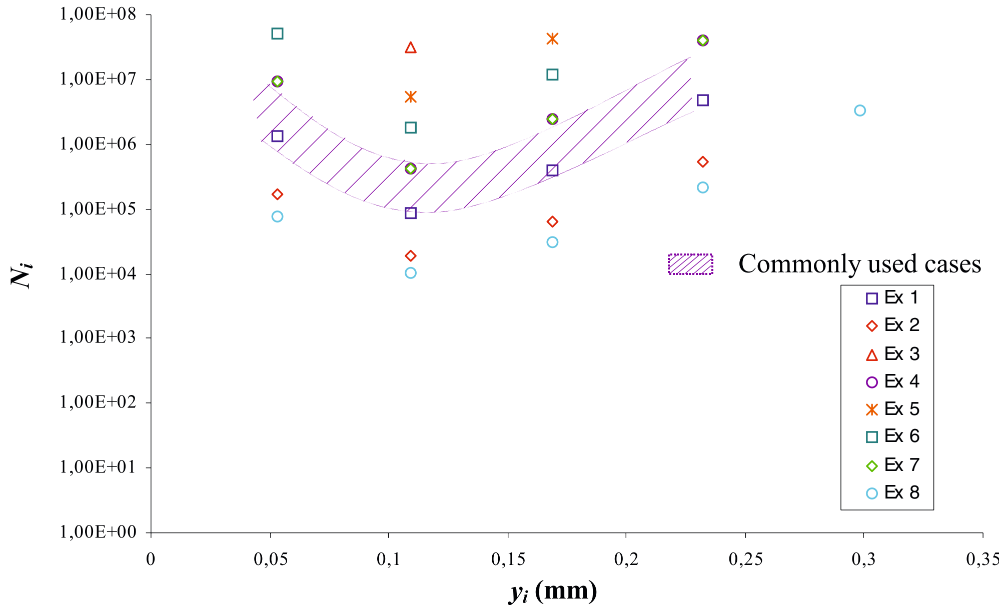

Fig. 8 Results: number of loading cycles when the initial crack appears at the observed material points (y_i) - Tresca comparative stresses, ε-N analysis method

The obtained loading cycle (regarding point B), corresponds well with the fact that, in most cases, the compressive stress on spur gear teeth occurred at the lowest point on a pinion tooth at which the full load was carried by a single pair of teeth. Theoretically, if one pair of teeth carries the full load and the position of loading is at point B, this is the worst-load condition for Hertzian contact stress corresponding to the worst-load position [7] for strength described in the preceding sections of the paper presented.

The values of the normalized stress cycles are higher under contact surface, which are influenced, in particular, by contact stresses and the coefficient of friction at the meshing points. In any case, at critical areas, the high contact pressure is combined with a significant amount of gear sliding velocities, which results in the frictional loading of the surface layer. On the basis of the defined contact loading cycles of meshing gears, considering teeth flanks loading, the fatigue analyses can then be performed.

3.2 Evaluation of fatigue crack initiation of teeth flanks

The strain-life method (ε-N) and its modified forms, presented in Section 2.3, was used, in the framework of the FEM program package MSC/FATIGUE [20], to determine the number of stress cycles N_i required for the fatigue crack initiation (starting of pitting damage process). The material parameters. $\sigma_f' = 1820 MPa$, $\varepsilon_f' = 0.65$, $b = -0.08$ *and* $c = -0.76$ were used for this purpose (Table 4).

The additional factors, influencing fatigue life, are: machine component size C_{size}, the type of loading C_{load} and

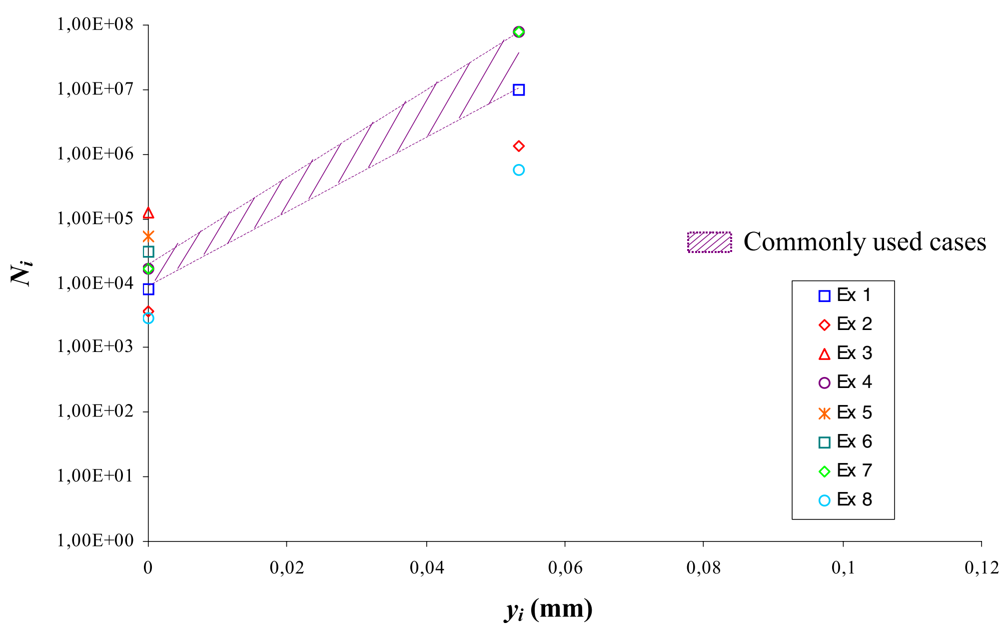

Fig. 9 Results: number of loading cycles when the initial crack appears at the observed material points (y_i) - maximal principal stresses σ_1, SWT analysis method

the effect of surface finish and treatment C_{surf}. Practically all fatigue failures start at the surface. So, it is evident that fatigue properties are very sensitive to surface conditions [7, 8]. The usual way to account for these effects is through the calculation and application of specific modifying factors that influence the actual fatigue (endurance) limit S_e for a real component (spur gears):

$$S_e = S'_e \cdot C_{surf} \cdot C_{size} \cdot C_{load}, \tag{15}$$

where S'_e is the measured material endurance limit.

The influence of surface finish on fatigue strength was considered using the surface finish factor C_{sur}, and R_a for different surface roughness (Table 2, Fig. 7). Surface finish is categorized by means of qualitative terms such as polished and machined, and the surface treatment is categorized by means of nitrided and shot peened, which provide the state of the residual stresses in a compressive layer and has the effect of decreasing the likelihood of fatigue failure.

The results of the number of loading cycles for crack initiation are presented in Figs. 8 and 9.

4 Results and discussion

In regards to the Tresca based loading cycles and the ε-N method for determination of initiation damage, the first damage appears at $(10^4 \div 3 \cdot 10^7)$ loading cycles (at approximately 0.12 mm under the observed inner point of a single teeth pair engagement; point B). In regards to the maximum principal stress σ_1 based on the loading cycles (Fig. 9) using the SWT method, the first damage appears at $(2,8 \cdot 10^3 \div 7,7 \cdot 10^7)$ loading cycles on the contact surface. The modified Coffin-Manson methods for the determination of crack initiation (presented in Section 2.3) are used, regarding the different nature of loading cycles (distribution of loading cycle due to different loading patterns or kinds of dynamic loading). Problems of mechanical treatment (examples 1–8 in Table 3) result in different numbers of critical loading cycles for crack initiation. Hatched areas on the diagrams presented (Figs. 8 and 9) are valid for the most frequently used gear pairs in practice. Thus, the average values of the critical number of loading cycles are in the range of $(1,2 \cdot 10^4 \div 7,1 \cdot 10^7)$ loading cycles, as defined in the proposed model.

Table 3 Cases studied and a description of the quality and and the method of treatment of the material

Cases studied (examples)	Case description
Ex. 1	Machined and shot peened: average
Ex. 2	Machined: average
Ex. 3	Machined and nitrided: good
Ex. 4	Polished
Ex. 5	Machined and nitrided: average
Ex. 6	Polished and shot peened
Ex. 7	Shot peened
Ex. 8	Machined: poor

Table 4 Fatigue material parameters for the generalized contact model

Parameter	Value
Modulus of elasticity	E=2.06·10^5 MPa
Poisson ratio	ν=0.3
Fatigue strength coefficient	σ'_f=1,820 MPa
Fatigue ductility coefficient	$\varepsilon'_f = 0.65$
Exponent of strength	b=−0.08
Fatigue ductility exponent	c=−0.76
Hardening exponent	n'=0.14
Strength coefficient	K'=2,259 MPa
Surface finish factor	C_{surf}=0.6÷0.9

However, in this paper, determination of fatigue crack initiation is defined with the aid of the continuum mechanics approach and the results are valid only for the approximately determined fatigue-damage initiation. Detailed analysis of such a phenomenon should also consider micromechanical approaches of material behaviour during fatigue process, which is, on the other hand, very difficult to achieve. Nevertheless, some particular effects on the service life of teeth flanks are considered via the parameters of surface treatment $\left(C_{surf}, R_a\right)$, material parameters taken from fatigue tests $(\sigma'_f, \varepsilon'_f, c, b)$ and geometrical and kinematical parameters of the equivalent model presented (Tables 1, 2, 3 and 4).

5 Concluding remarks

Gears are very specific parts of machines, and a lot of field experience and testing has been done for over a century; the standards for calculation, consequently, are more or less detailed (AGMA, ISO, DIN). However, gears are subjected to fatigue loads (surface contact fatigue and bending fatigue). The kind of fatigue, which is the subject of the presented work, is contact fatigue of gear teeth flanks. The paper presents a general computational model for the determination of initiation fatigue loading cycles in meshing gears. An equivalent contact model of a cylinder and flat surface is used for the simulation of contact fatigue crack initiation under conditions of rolling and sliding contact loading of meshing gears. The material model is assumed to be homogenous, without imperfections and/or inclusions, and elastic shakedown is considered. The modified Coffin-Manson method, in the framework of finite element analysis (FEM), is used for iterative analyses of contact fatigue crack initiation. The number of loading cycles and places (on/under the contact surface) for contact fatigue are presented.

Generally, regardless of the stress component selected, the number of loading cycles required for initial fatigue damages is in the range of $N_i = (10^4 \div 10^7)$ and where (on the contact surface or subsurface) the contact fatigue damage first occurs mostly depends on the coefficient of friction, material parameters and contact geometry.

Although in final damage by pitting where small cracks form in the tooth surface and then grow to the point where small, round bits of metal break out of the tooth surface (Fig. 1), the whole fatigue process should be estimated with the total service life, i.e. Eq. (1).

Drawbacks of the proposed model are certainly the unconsidered residual stresses, elastic-hydrodynamic lubrication conditions and material non-homogeneity. Also, some additional experimental work should be provided for this purpose. These are topics for further improvement of the presented computational model.

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